



Copyright SBE Builders

Tangent handrailing wreath developed from both books. A Treatise on StairBuilding & Handrailing is theoretically geometrical correct, but you need A Simplified Guide to Custom StairBuilding and Tangent Handrailing to understand W & A Mowat.

BOOKS PUBLISHED TREATING ON STAIR-BUILDING.

THE following as a complete list and dates, as far as ascertained, of publications in the English language that either treats partially or wholly of stair-building and hand-railing: DATE.

- 1693. Moxon, "Mechanical Exercises."
- 1725. Halfpenny, "Art of Sound Building."
- 1735. Francis Price, "British Carpenter."
- 1738. Batty Langley, "Builder's Complete Assistant."
- 1750. Abraham Swan, "Architect."
- 1792. Peter Nicholson, "Carpenter's Guide."
- 1813. Peter Nicholson, New "Carpenter's Guide."
- 1826. M. A. Nicholson, "Carpenter, Joiner, and Builder's Companion."
- 1864. Joshua Jeays, "Orthogonal System of Hand-railing."
- 1873. Newland's "Carpenter and Joiner's Assistant."

The above are all English publications.  
The following are all-or have been-published the United States :

- DATE.
- 1844. R. G. Hatfield, "The American House-carpenter."
  - 1845. Simon De Graff, "The Modern Geometrical Stair-builder's Gui"
  - 1849. Cupper's "Hand-railing."
  - 1856. Robert Riddell's " Hand-railing."
  - 1858. Perry's " Hand-railing."
  - 1859. Esterbrook & Monckton's "American Stair-builder."
  - 1872. Monckton's "National Stair-builder."
  - 1873. Wm. Forbes's" The Sectorian System of Hand-railing."
  - 1875. Gould's "Hand-railing."
  - 1886. Frank O. Cresswell's Hand-railing and Staircasing."
  - 1888. Monckton's "Stair-building in its Various Forms, and One-plane Method of Drawing Face moulds and Unfolding the Centre Line of Wreaths."
  - 1889. John V. H. Secor, "Nonpareil System of Hand-land-railing."

All drawings on this page are from  
Stair Builders' Guide  
By Morris Williams  
1914

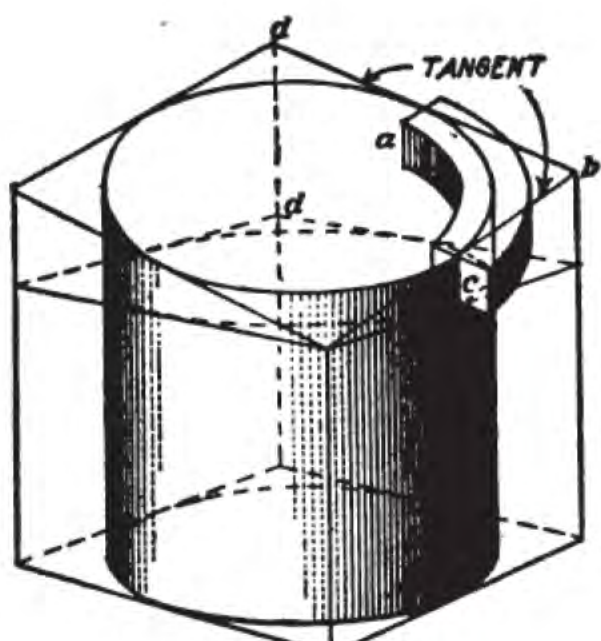
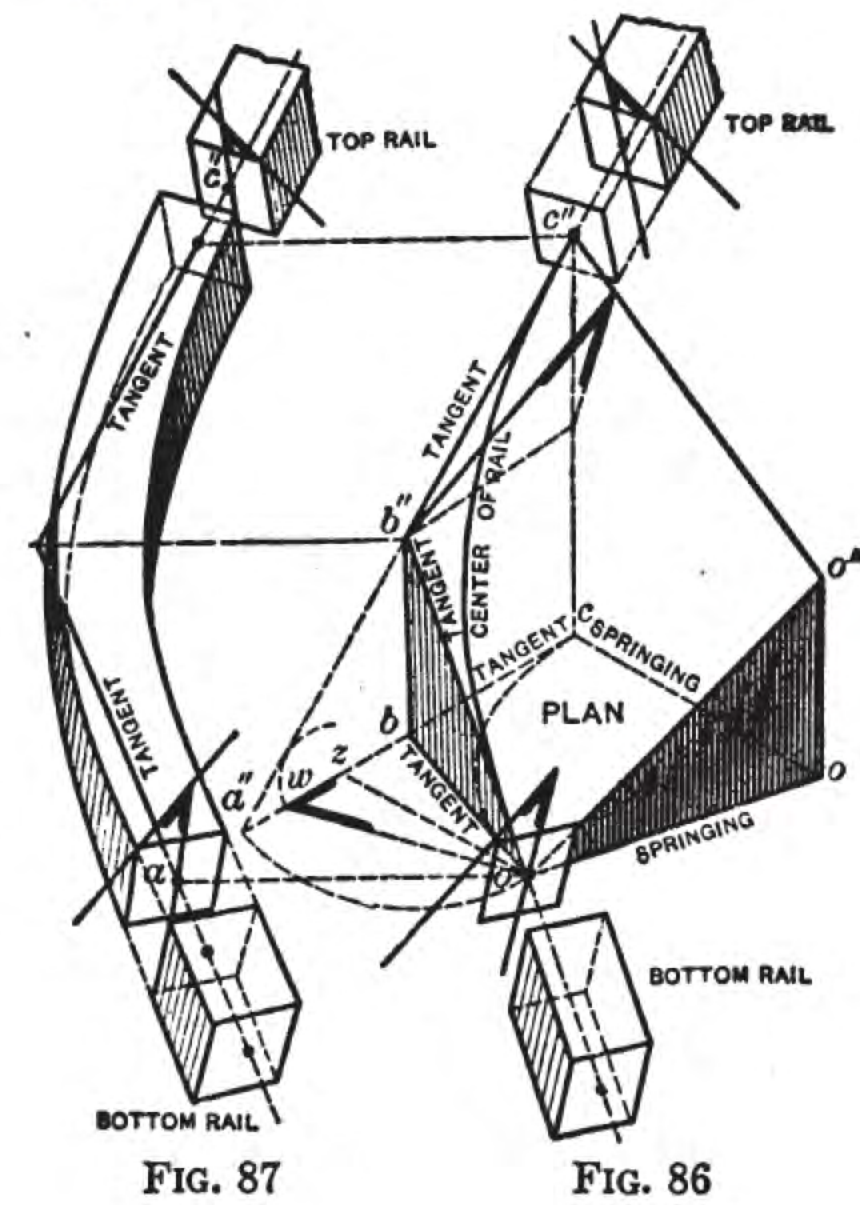
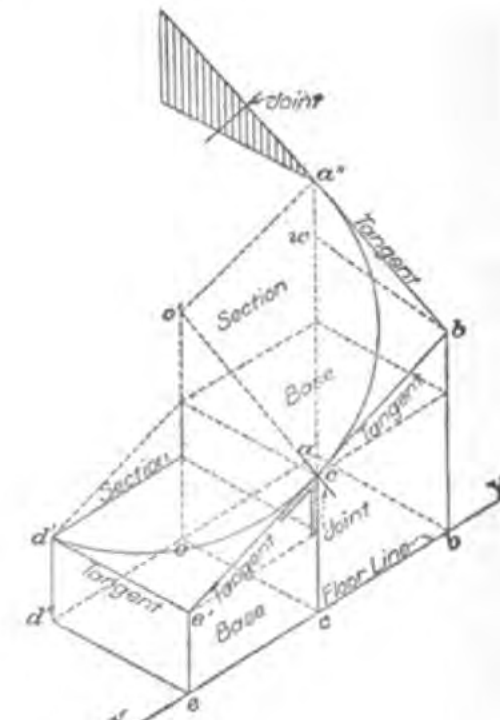
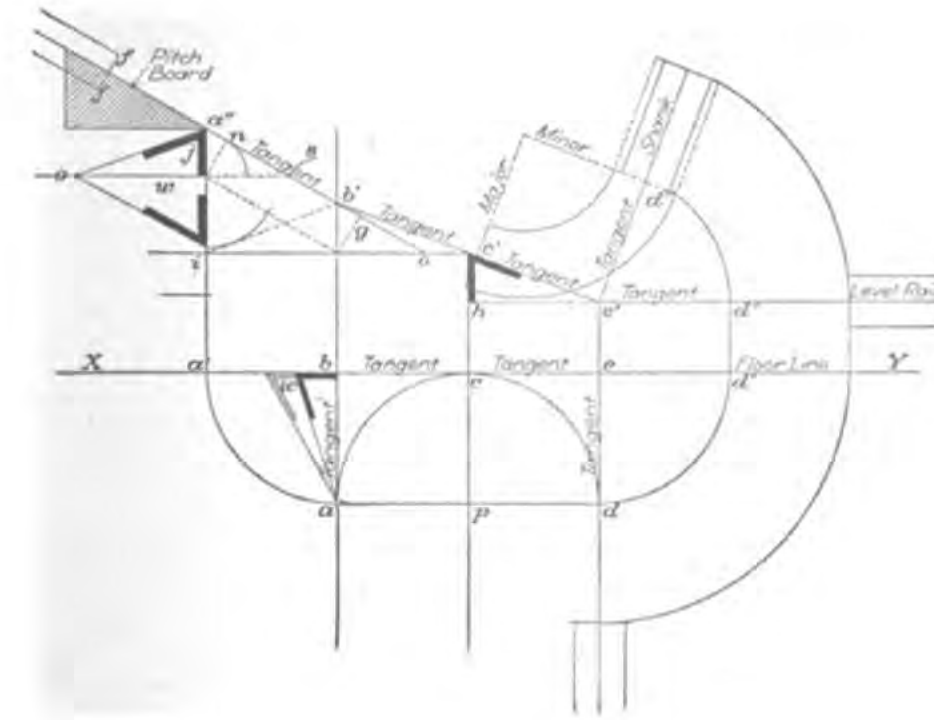
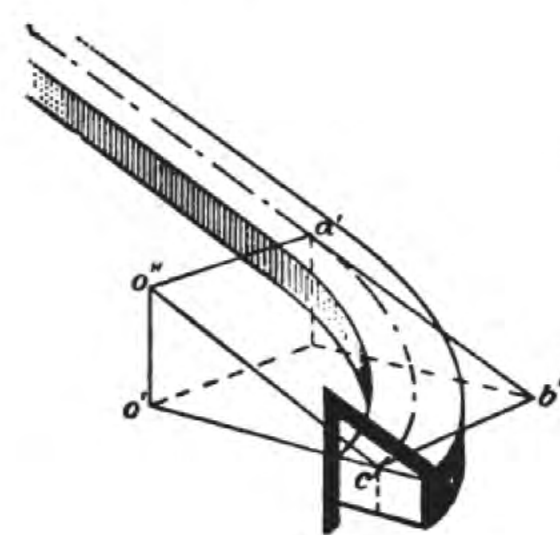


FIG. 48



Books on understanding the tangent handrail system.  
Treatise On Stairbuilding and Handrailing  
1900  
W & A Mowat

A Simplified Guide to Custom Stairbuilding and Tangent Handrailing  
1994  
George R. Di Christina

Stair builders' guide: a treatise on the construction of straight flight, platform, cylindrical and elliptical [!] stairs, explaining the theory and practice so the average building mechanic may understand it, with examples of work ranging from the simplest to the most complex forms  
1914  
Morris Williams  
<http://books.google.com/books?id=4350AAAAYAAJ&ots=gaoLPNPPmJ&dq=Stair%20builders%20guide%20by%20Morris%20Williams&pg=PP1#v=onepage&q&f=false> <<http://books.google.com/books?id=4350AAAAYAAJ&ots=gaoLPNPPmJ&dq=Stair%20builders%20guide%20by%20Morris%20Williams&pg=PP1>>

Building age, Volume 40  
Morris Williams  
[http://books.google.com/books?id=ko\\_AAAAYAAJ&lpg=RA6-PA526&ots=vxAtasq85K&dq=peter%20nicholson%20tangent%20handrailing&pg=RA6-PA526#v=onepage&q=peter%20nicholson%20tangent%20handrailing&f=false](http://books.google.com/books?id=ko_AAAAYAAJ&lpg=RA6-PA526&ots=vxAtasq85K&dq=peter%20nicholson%20tangent%20handrailing&pg=RA6-PA526#v=onepage&q=peter%20nicholson%20tangent%20handrailing&f=false) <[http://books.google.com/books?id=ko\\_AAAAYAAJ&lpg=RA6-PA526&ots=vxAtasq85K&dq=peter%20nicholson%20tangent%20handrailing&pg=RA6-PA526](http://books.google.com/books?id=ko_AAAAYAAJ&lpg=RA6-PA526&ots=vxAtasq85K&dq=peter%20nicholson%20tangent%20handrailing&pg=RA6-PA526)>

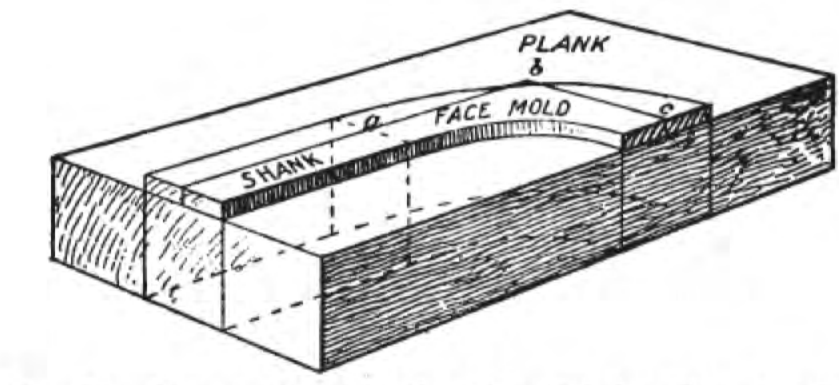


FIG. 75.—How to Cut Out the Material for the Wreath From the Plank.

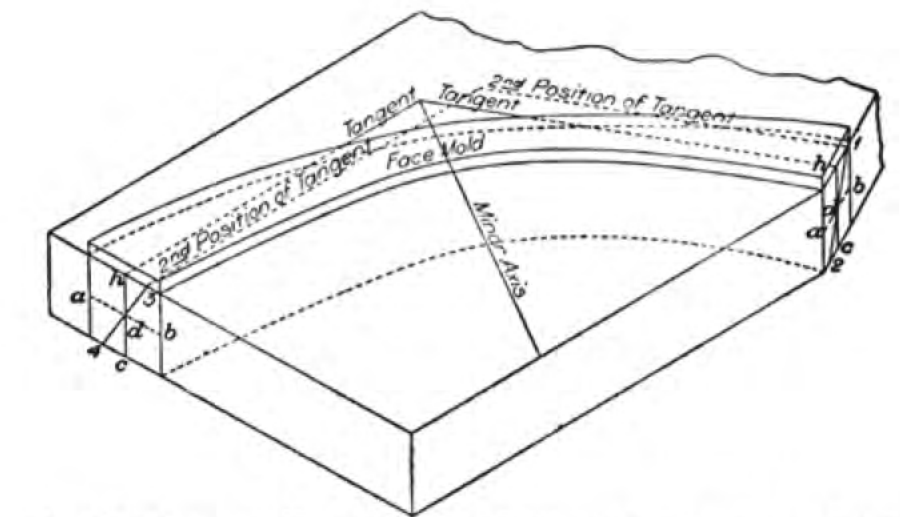
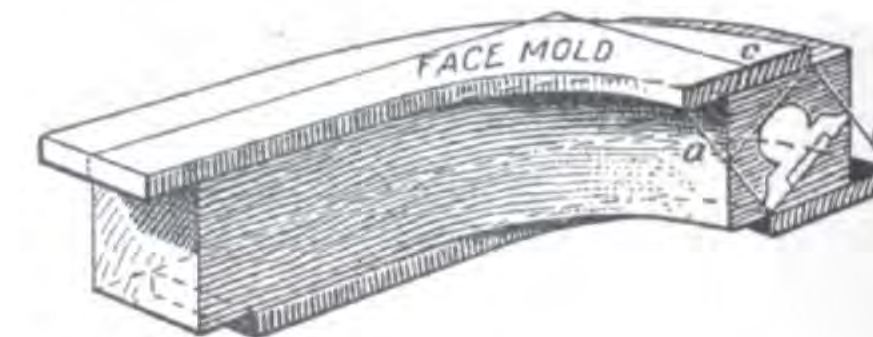


FIG. 77.—Isometric View, Showing Method of Applying Face Mold to Plank for Purposes of Cutting the Material for the Wreath.

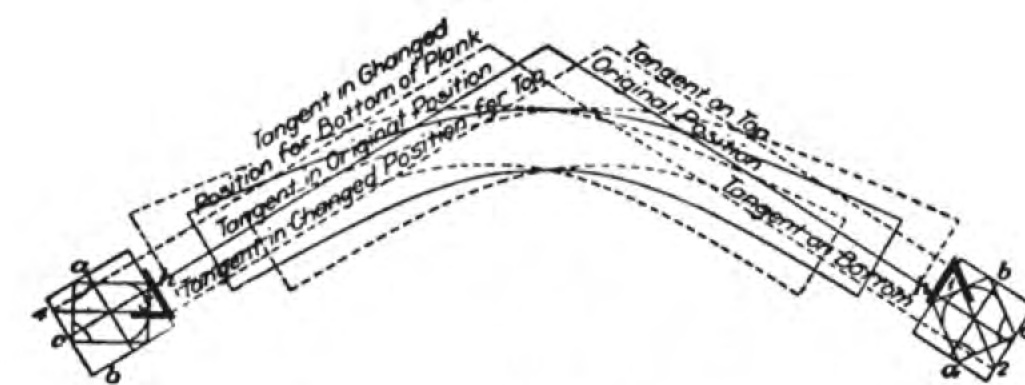


FIG. 78.—Diagram Illustrating Method of Applying Face Mold to Trace the Outlines of the Wreath Preparatory to Squaring its Verticle Sides.

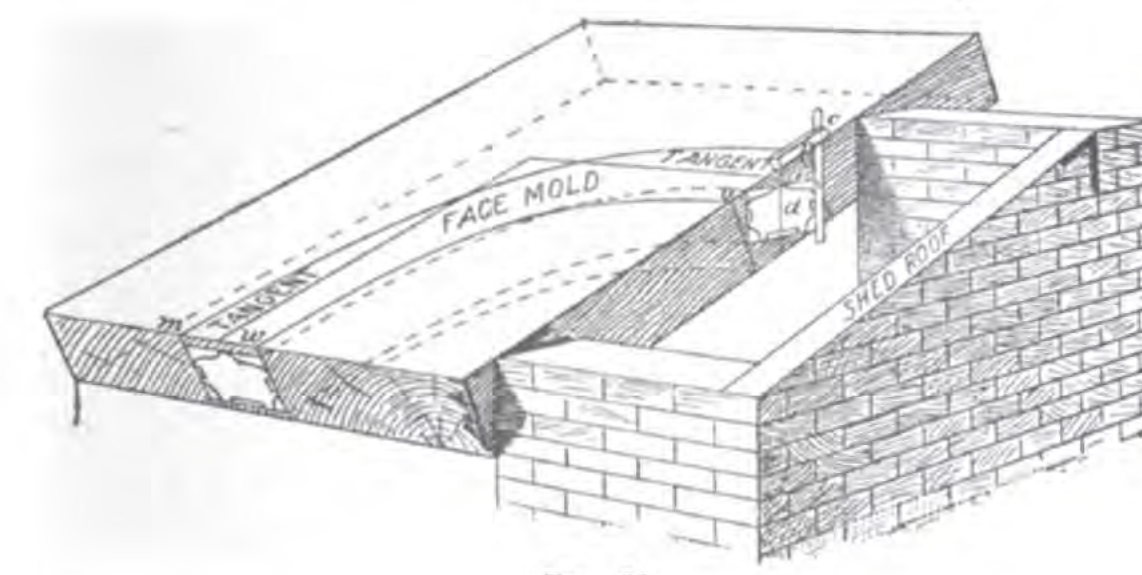
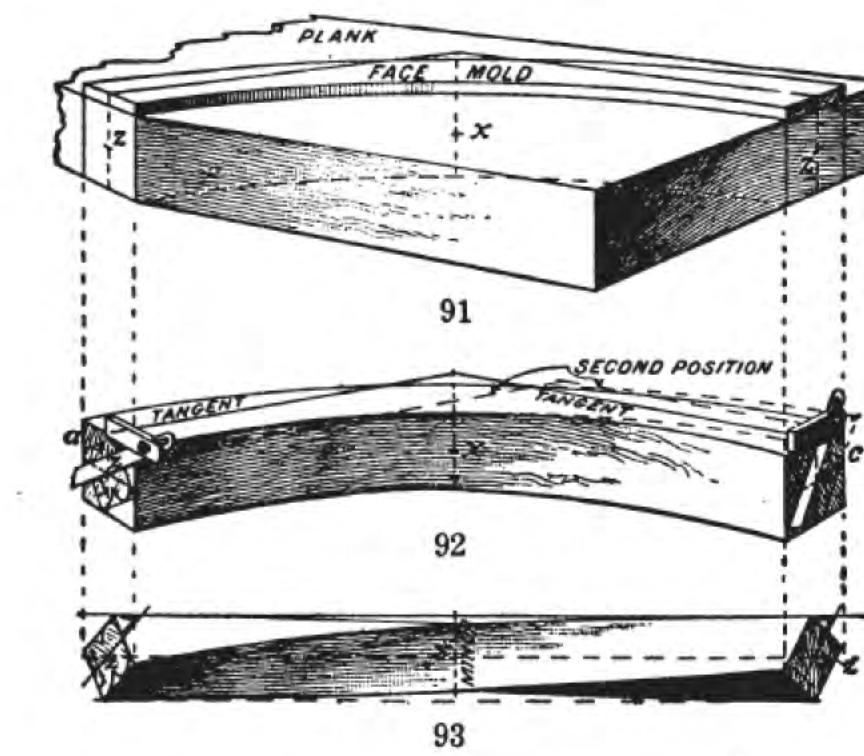


FIG. 59

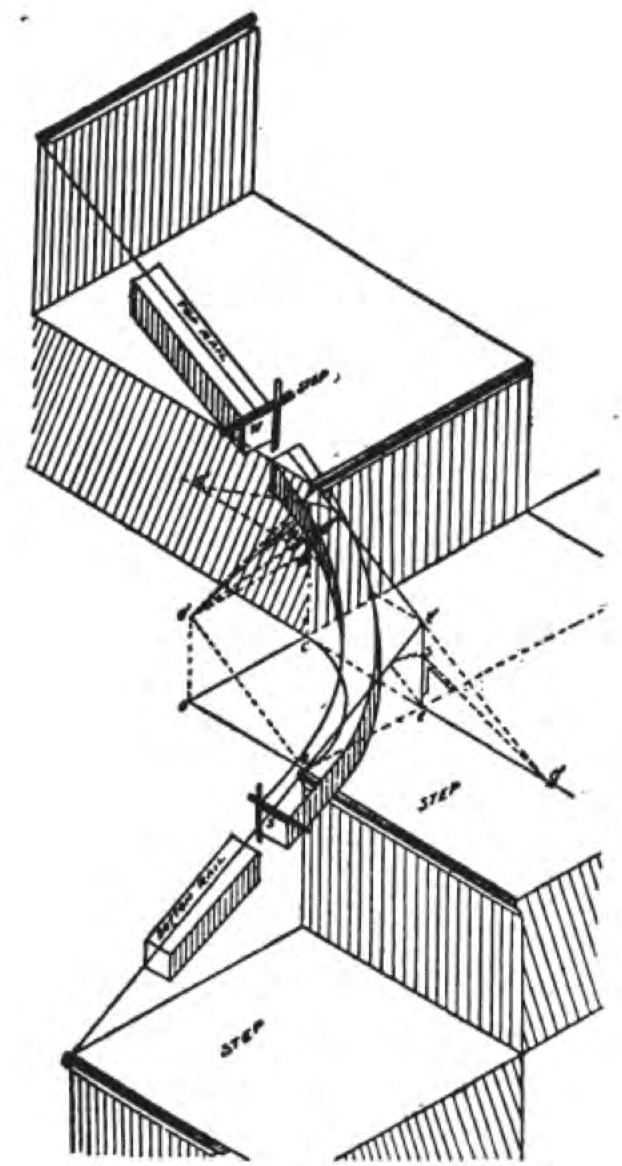


FIG. 90

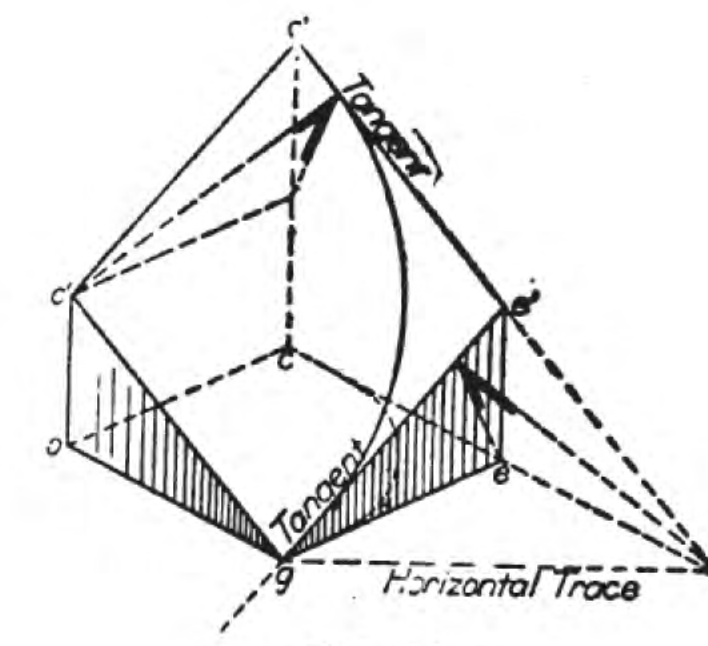


FIG. 89

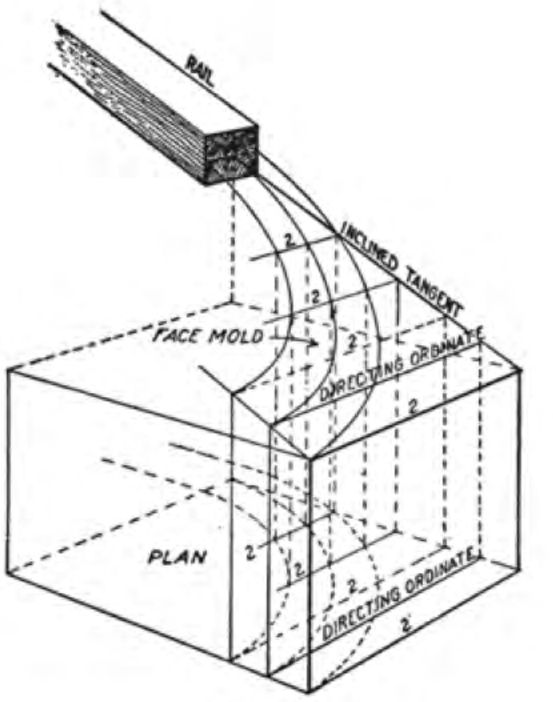
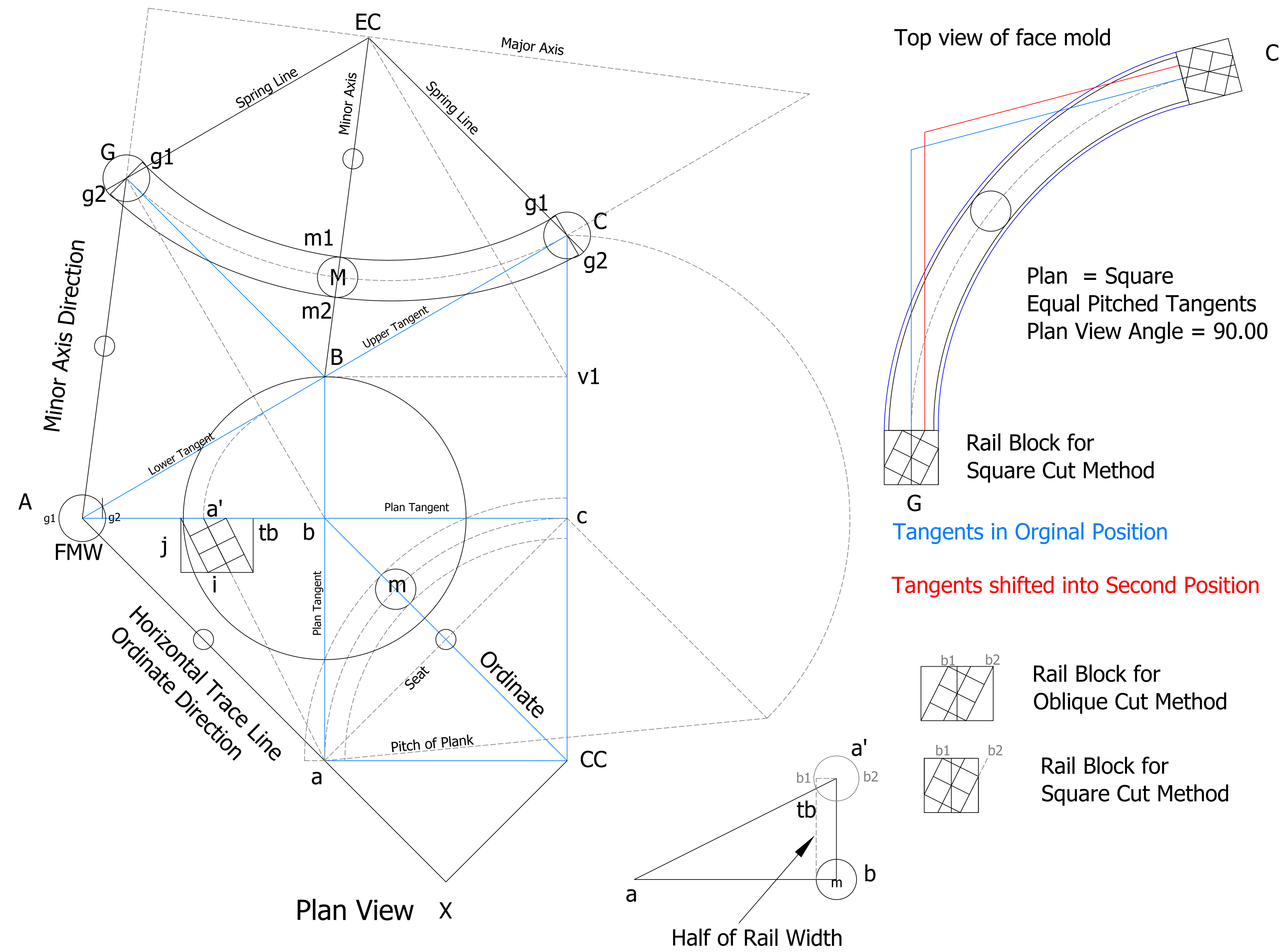


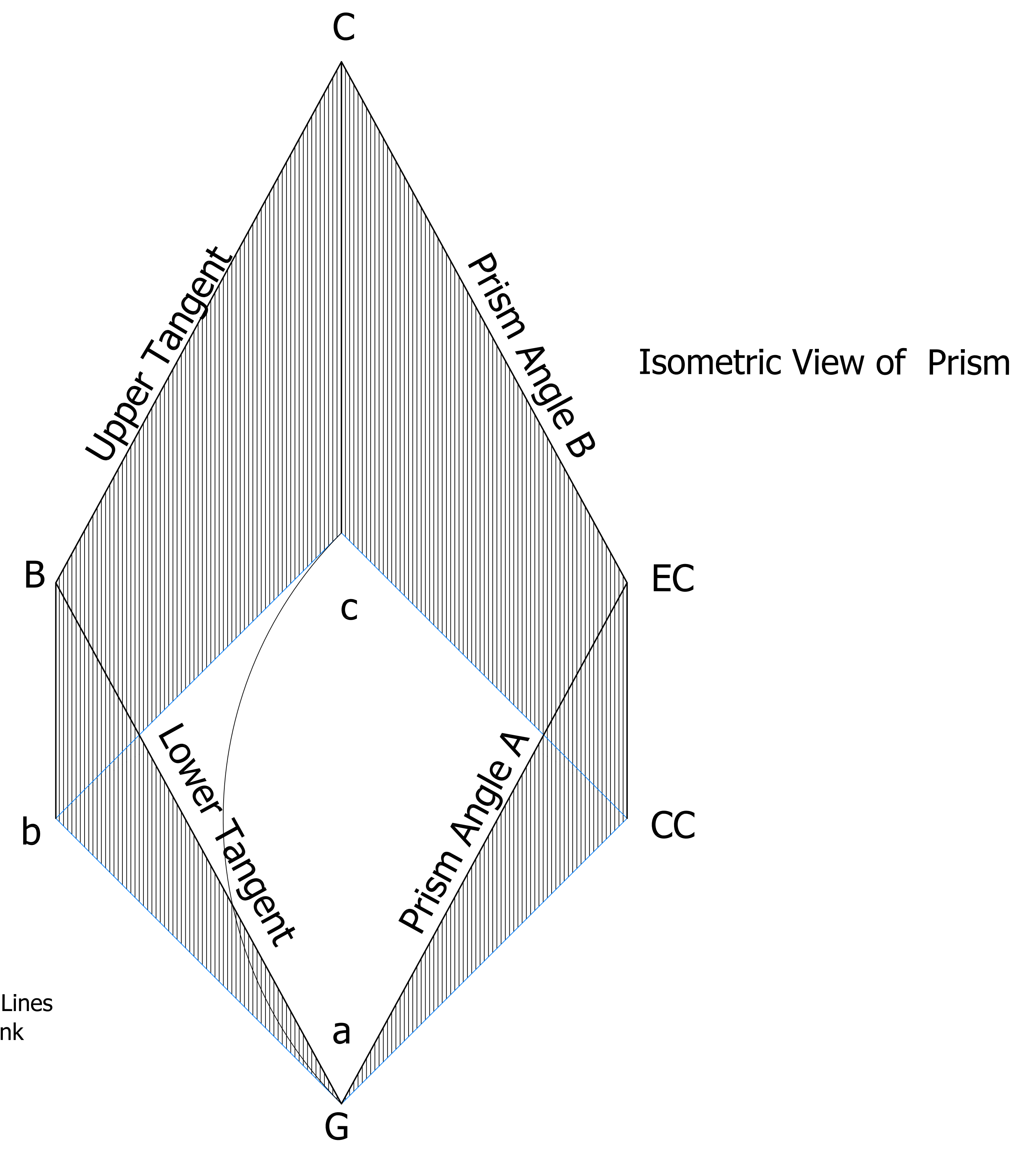
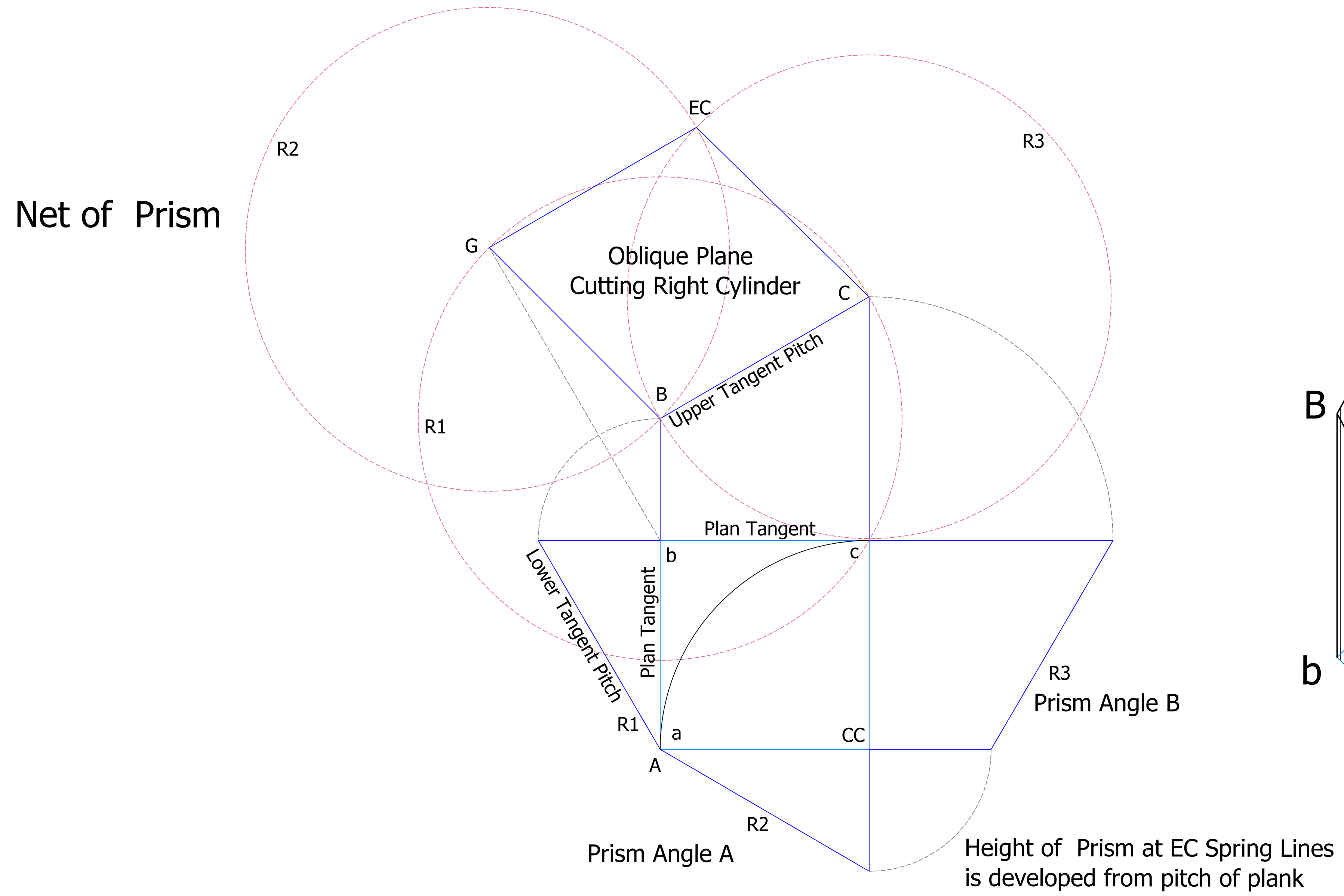
FIG. 69

Plate 1

Quarter-Circle Plan with Equally Pitched Tangents

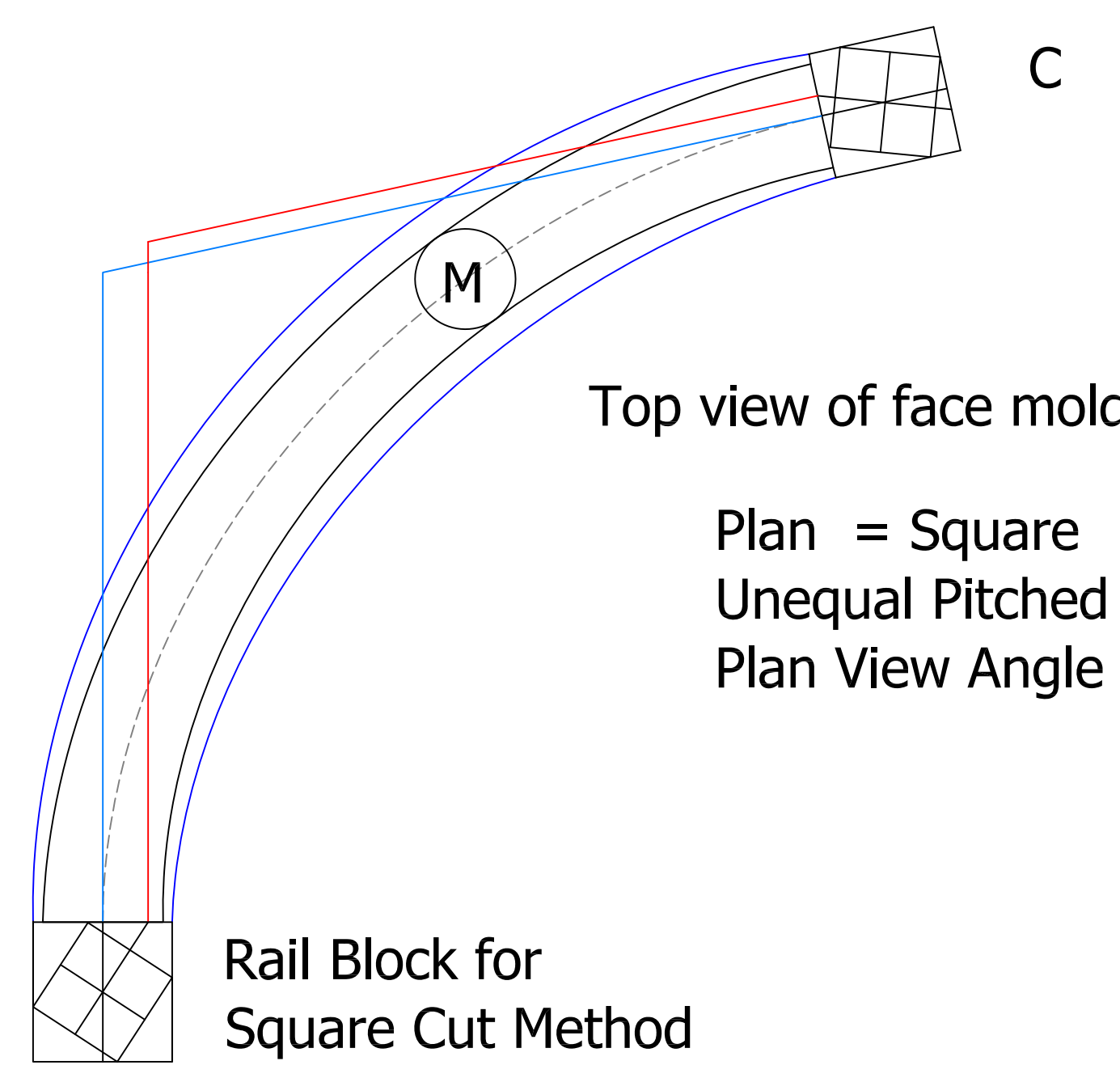
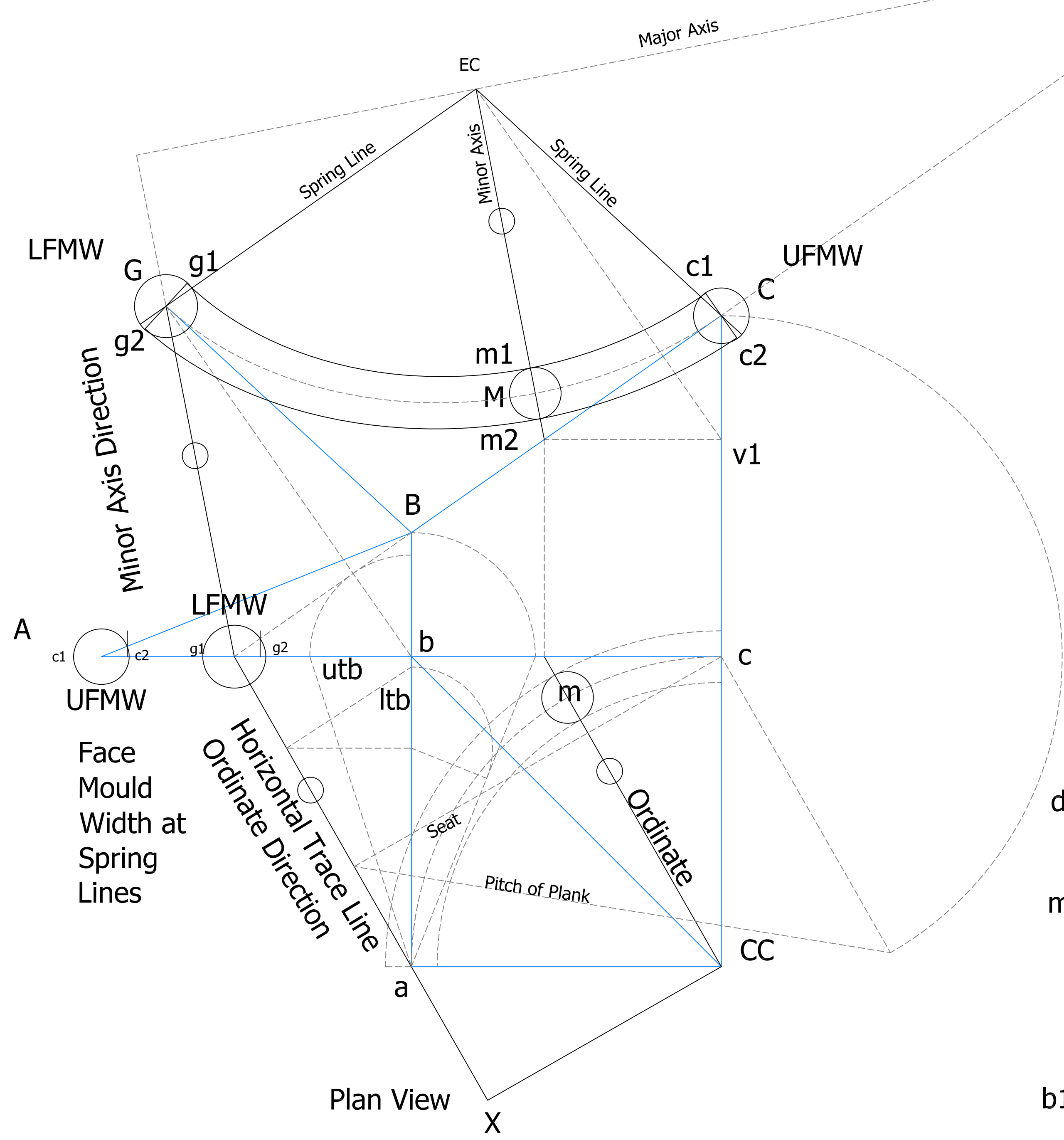


**Plate 1.1** Quarter-Circle Plan with Equally Pitched Tangents



**Plate 2**

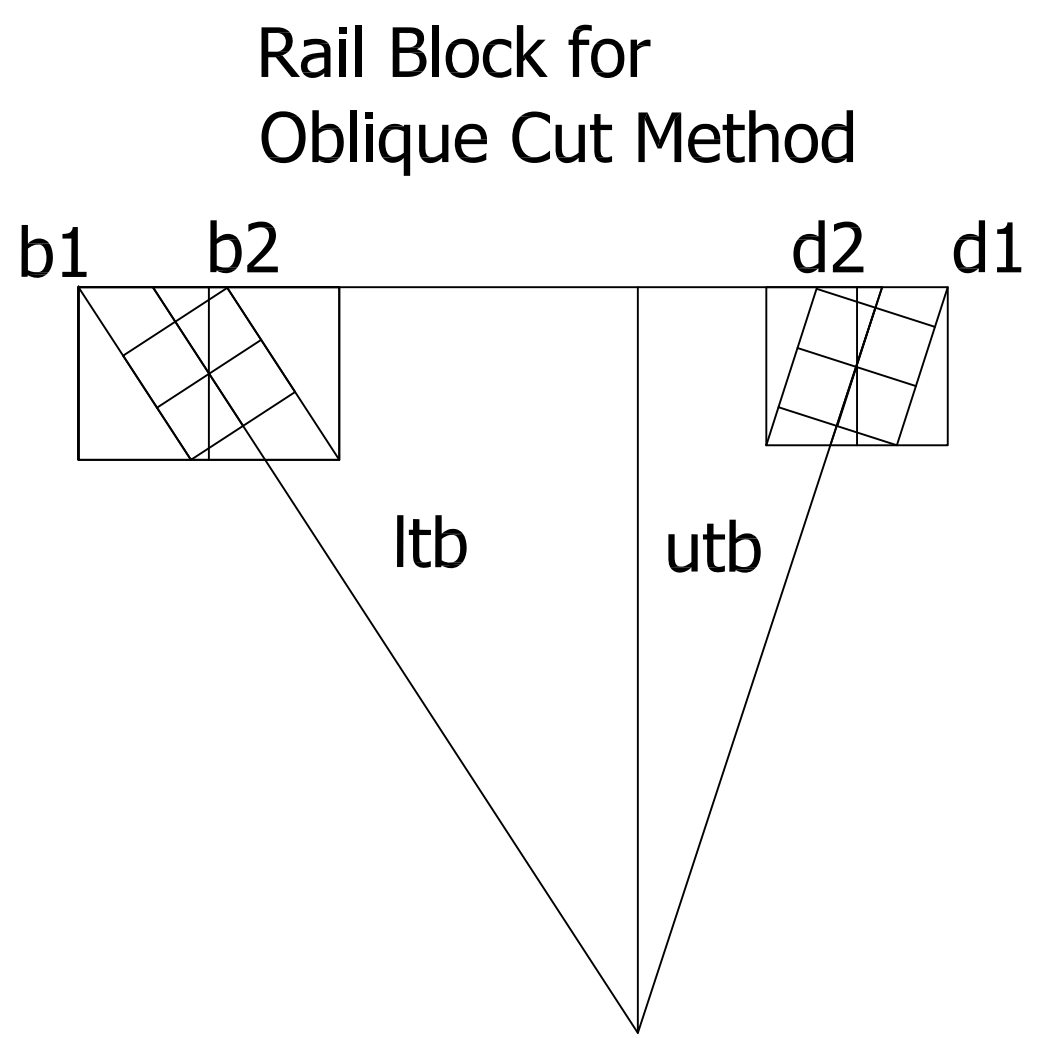
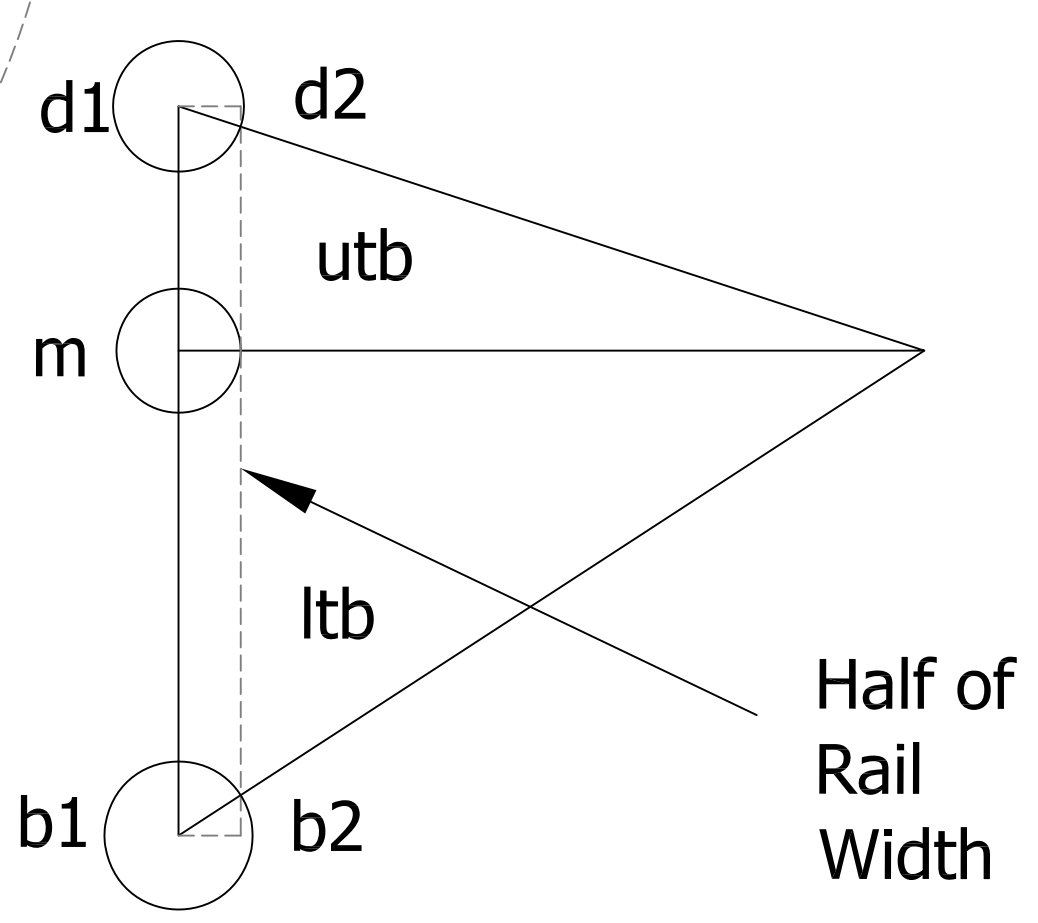
Quarter-Circle Plan with Short Lower Pitched Tangent



Top view of face mold  
 Plan = Square  
 Unequal Pitched Tangents  
 Plan View Angle = 90.00

Tangents in Original Position

Tangents shifted into Second Position

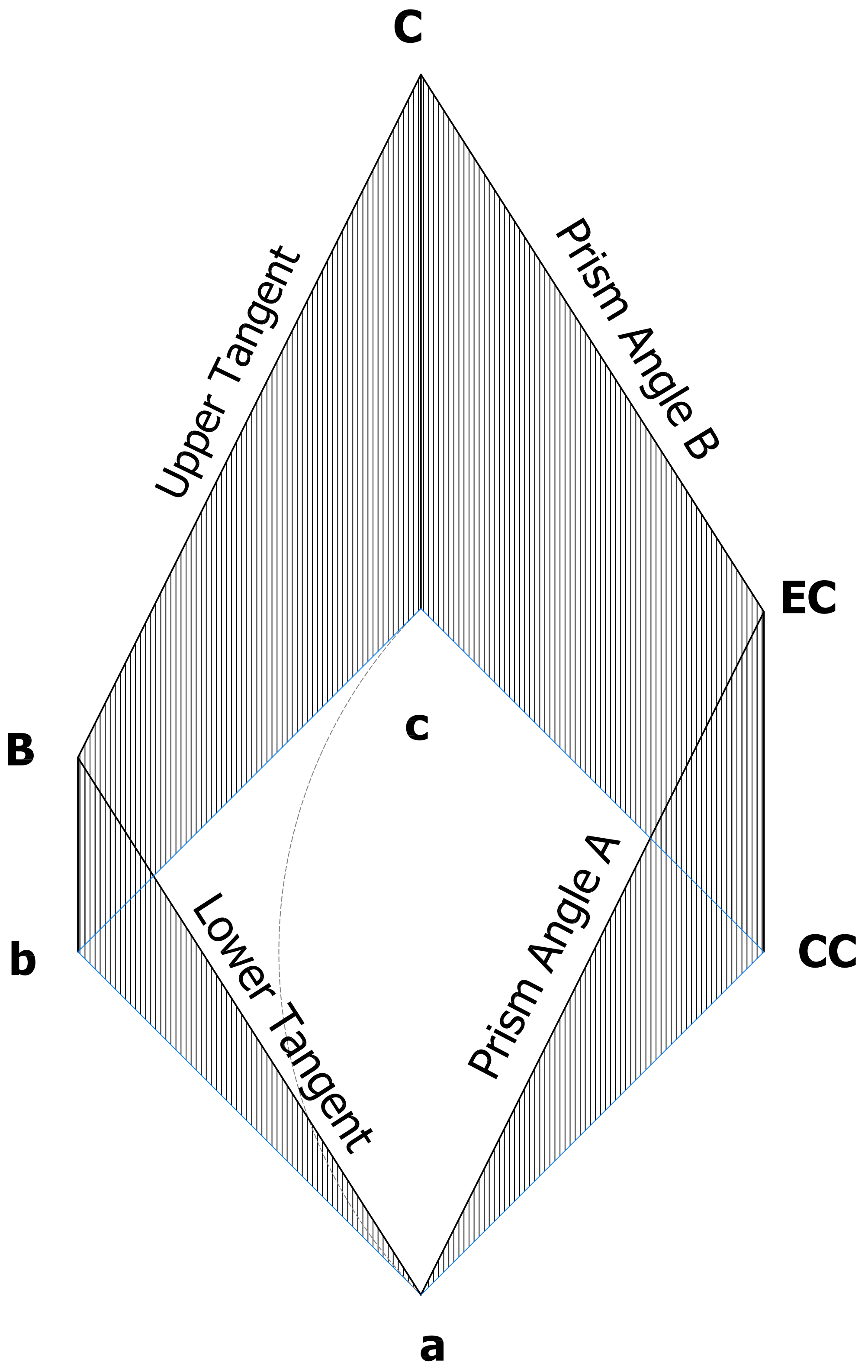
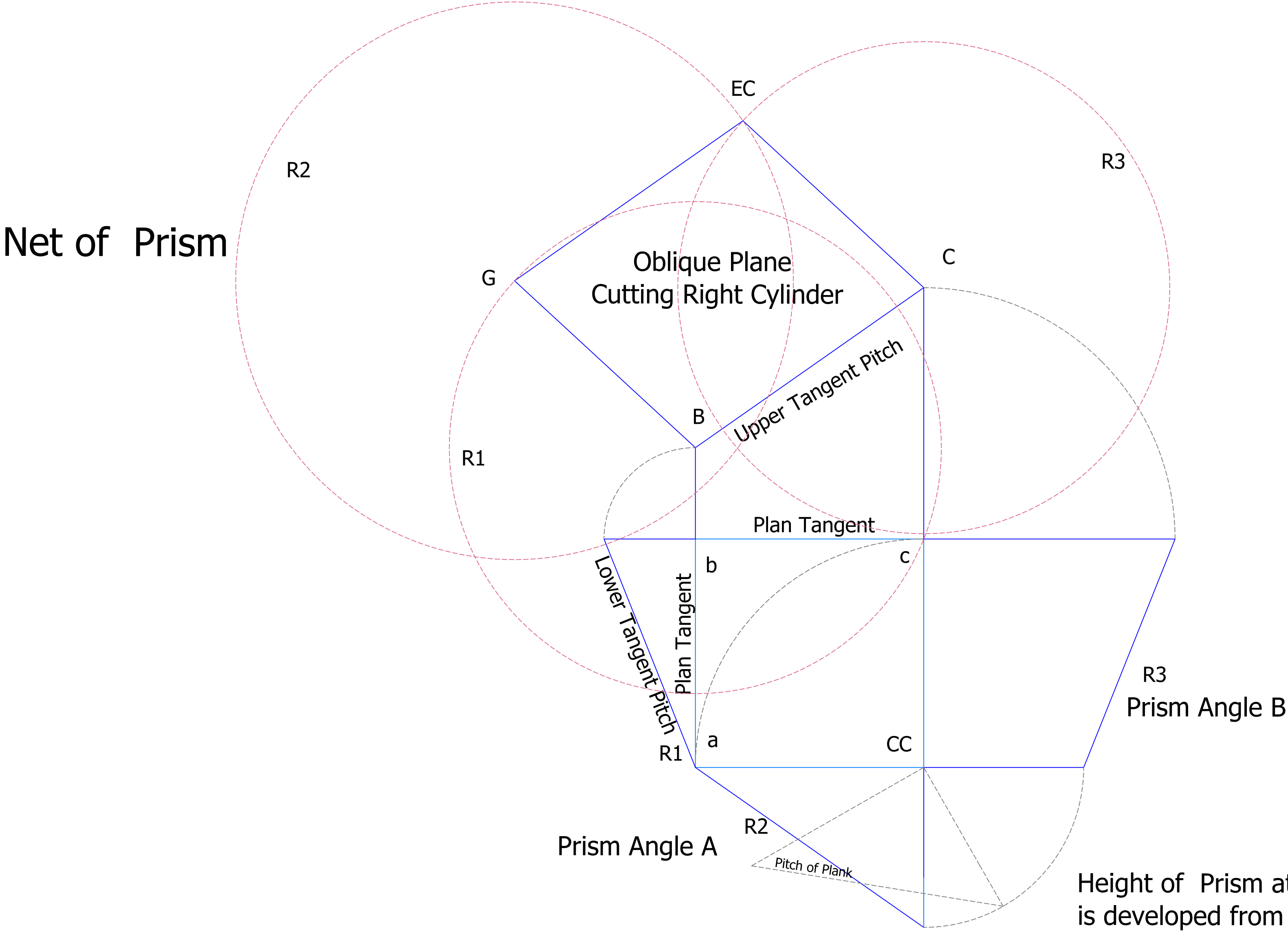


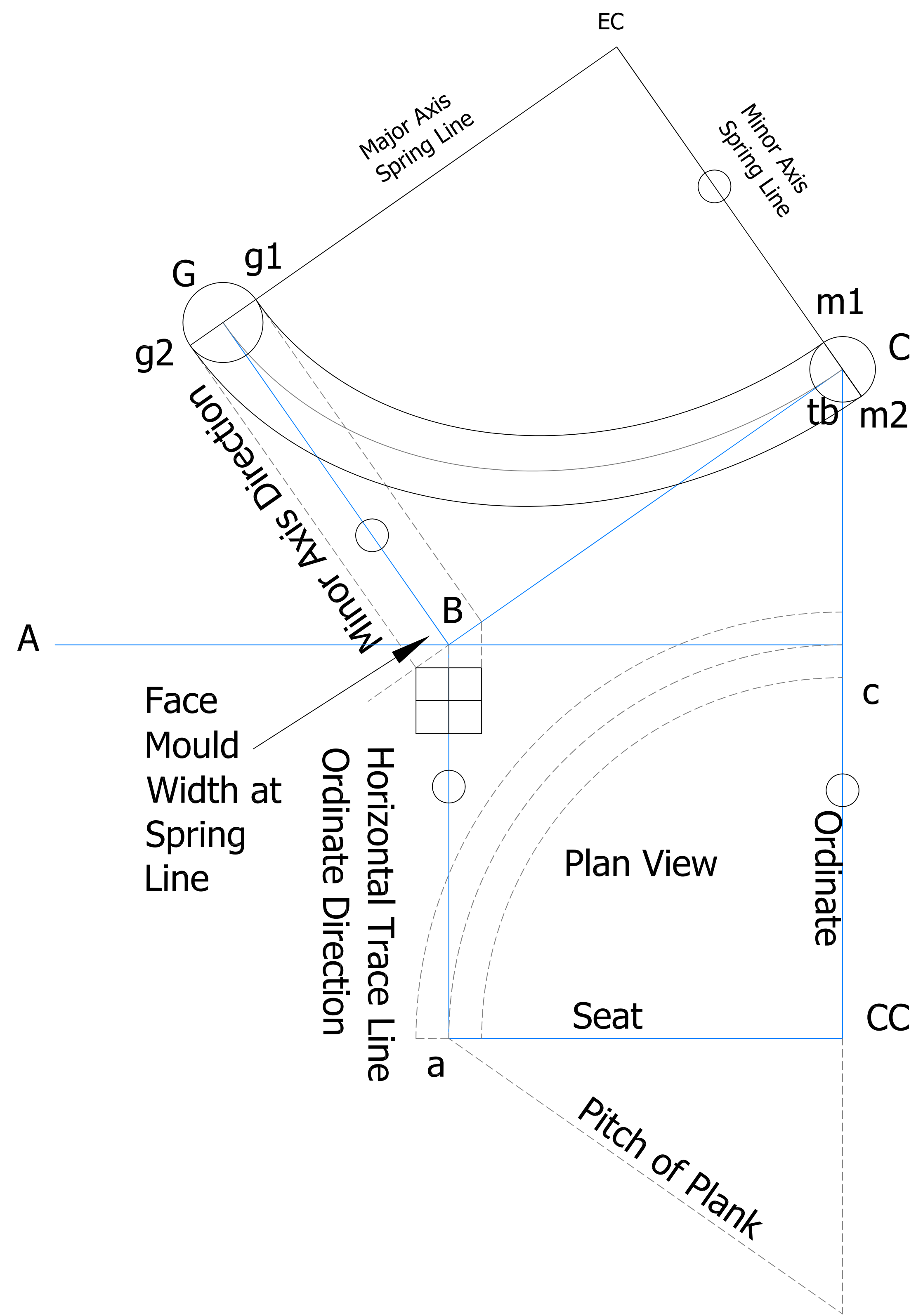
Rail Block for Oblique Cut Method

Half of Rail Width

Plate 2.1

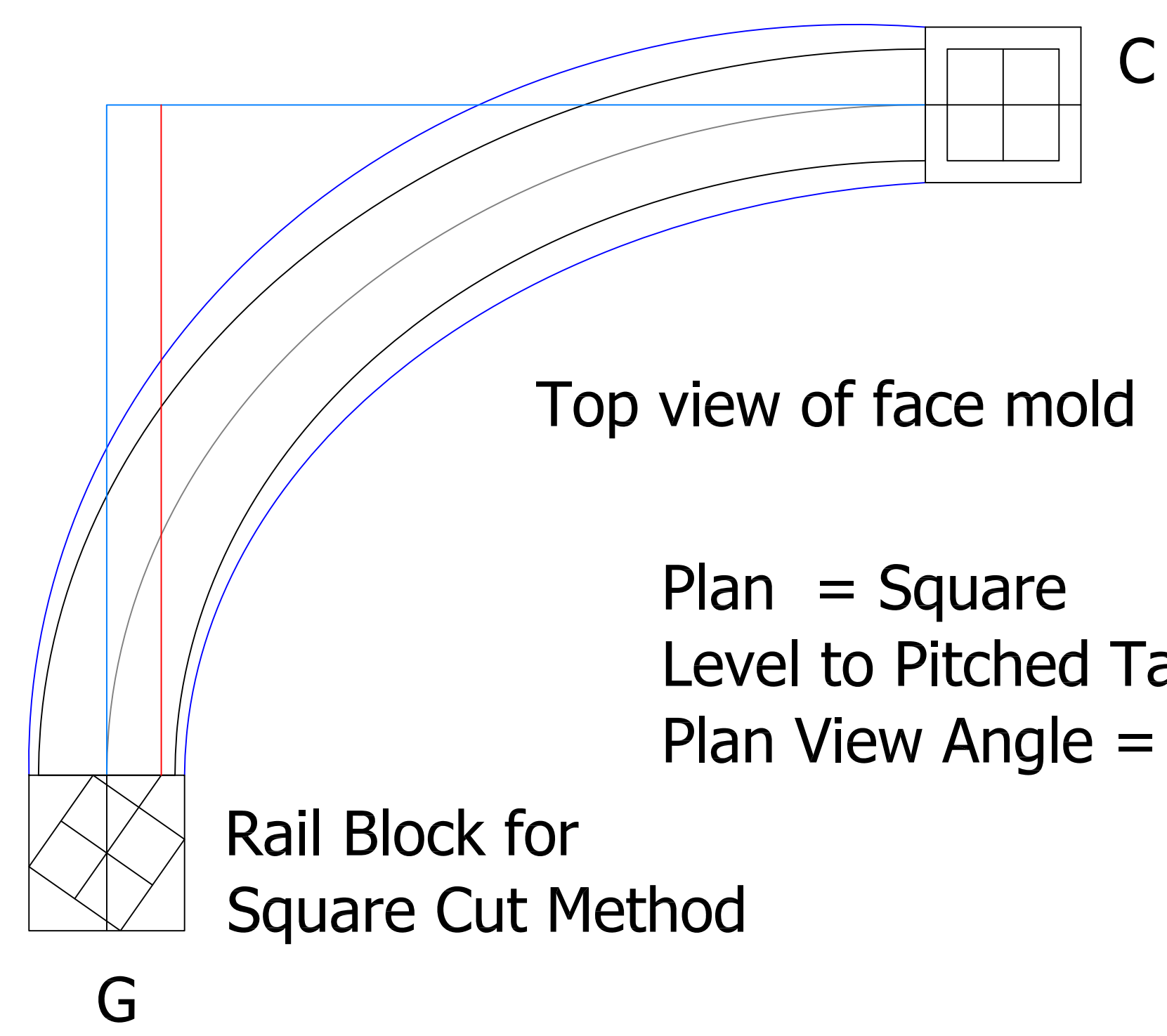
Quarter-Circle Plan with Short Lower Pitched Tangent





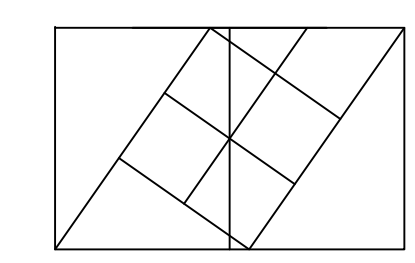
# Plate 3

Quarter-Circle Plan The Upper Tangent is Pitched, the Lower Tangent is Level



Tangents in Original Position

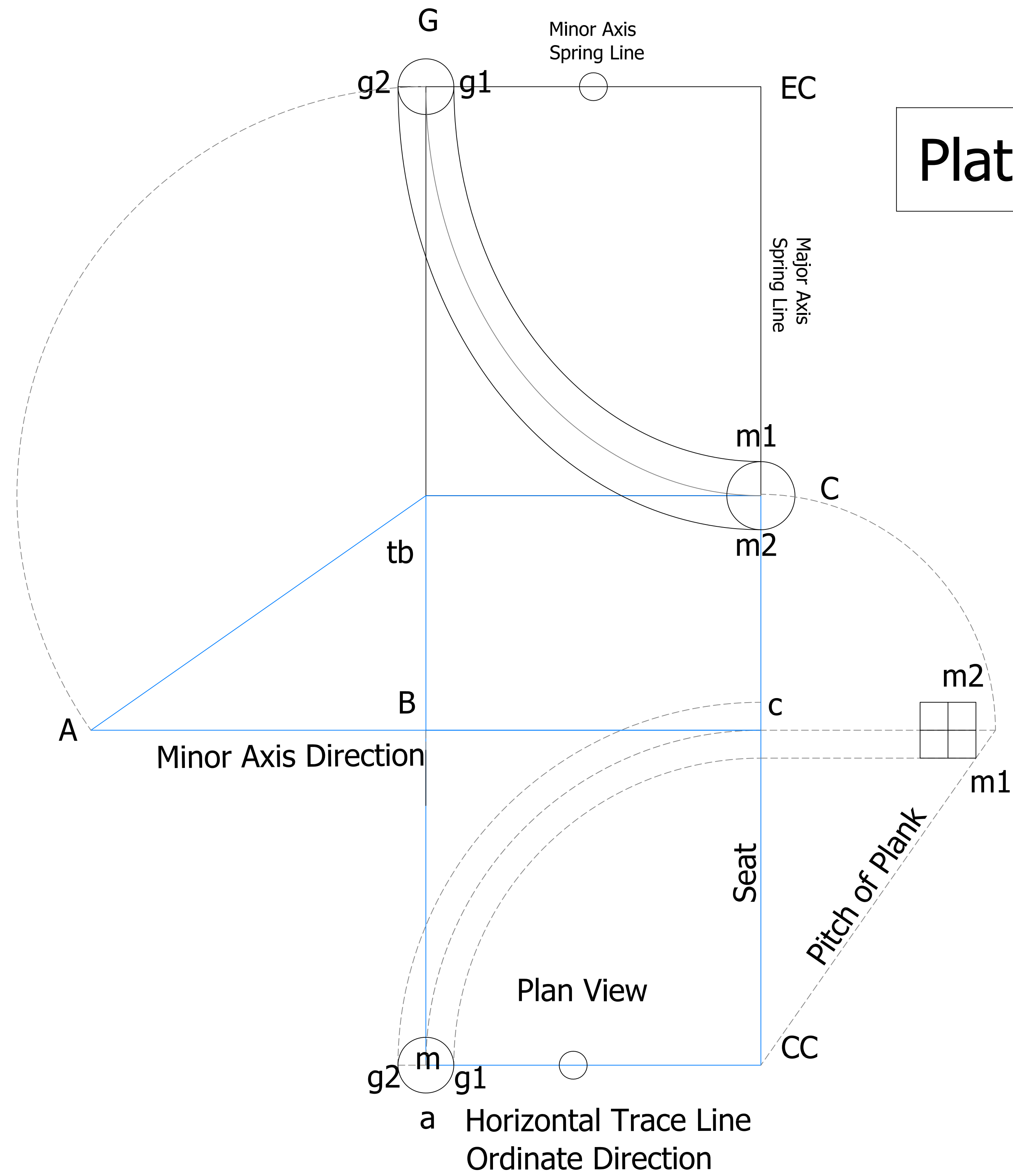
Tangents shifted into Second Position



Rail Block for Oblique Cut Method

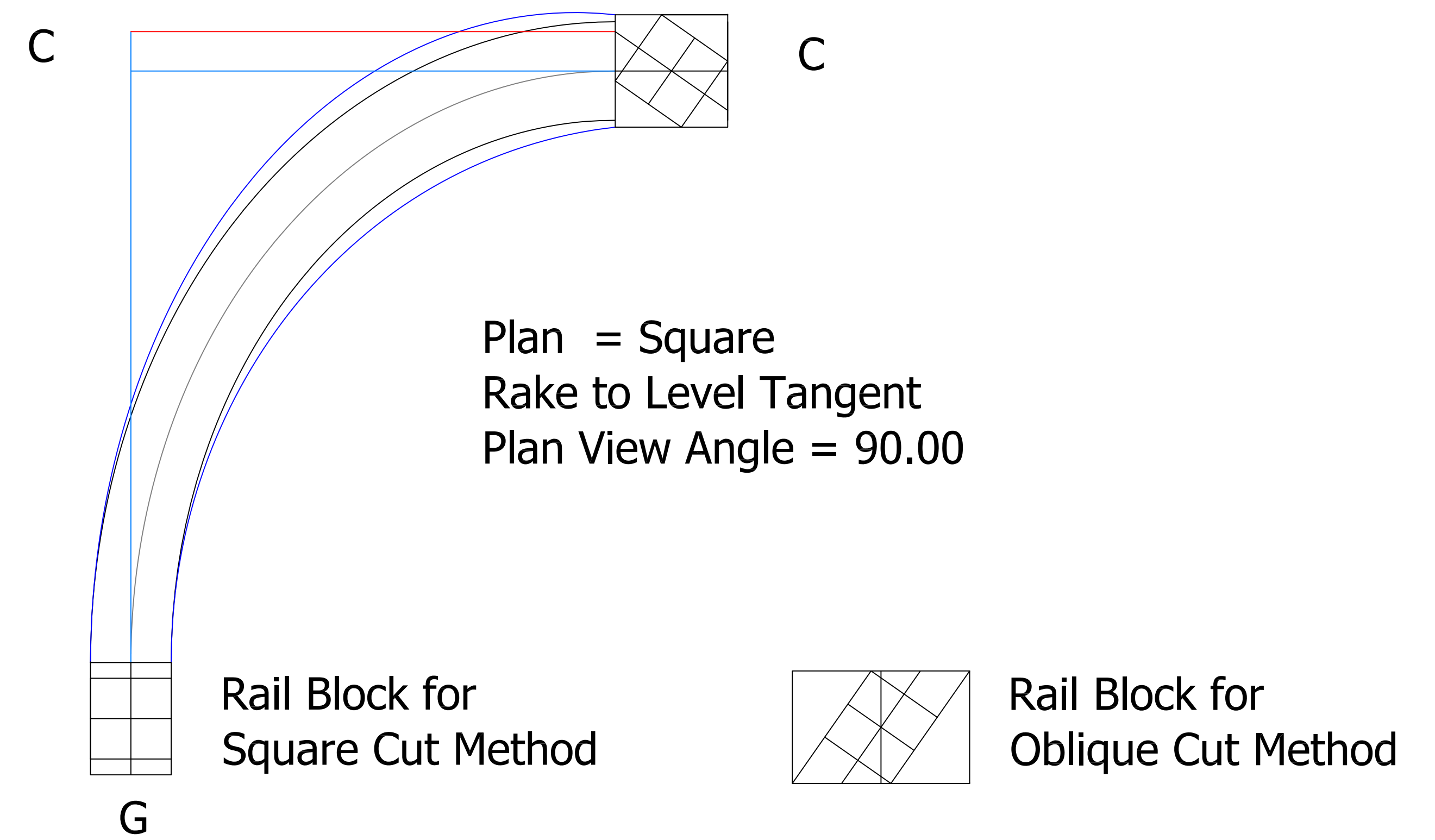






**Plate 4**

Quarter-Circle Plan The Upper Tangent is Level, the Lower Tangent is Pitched



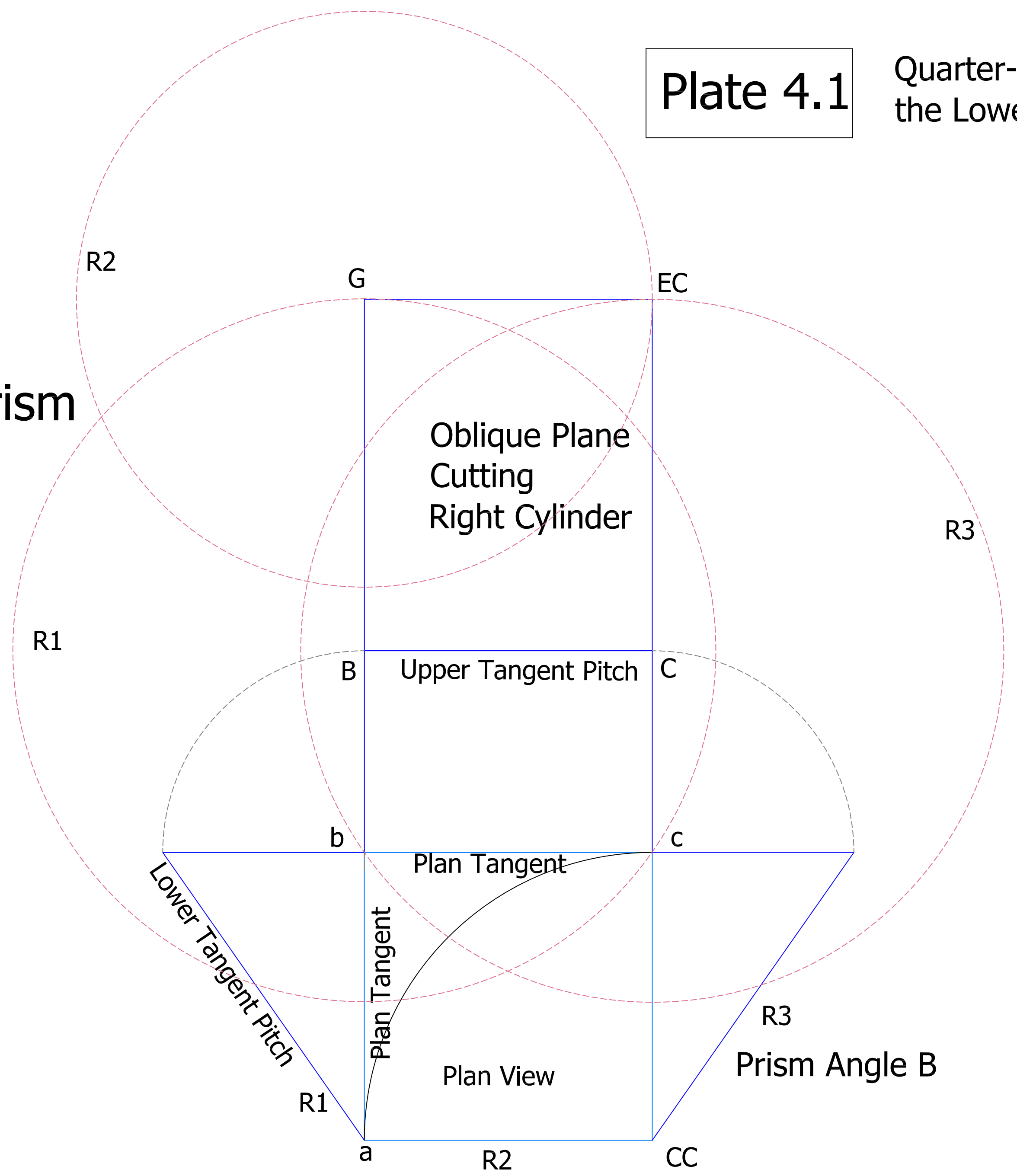
Tangents in Original Position

Tangents shifted into Second Position

**Plate 4.1**

Quarter-Circle Plan The Upper Tangent is Level,  
the Lower Tangent is Pitched

Net of Prism



Oblique Plane  
Cutting  
Right Cylinder

Upper Tangent Pitch

Plan Tangent

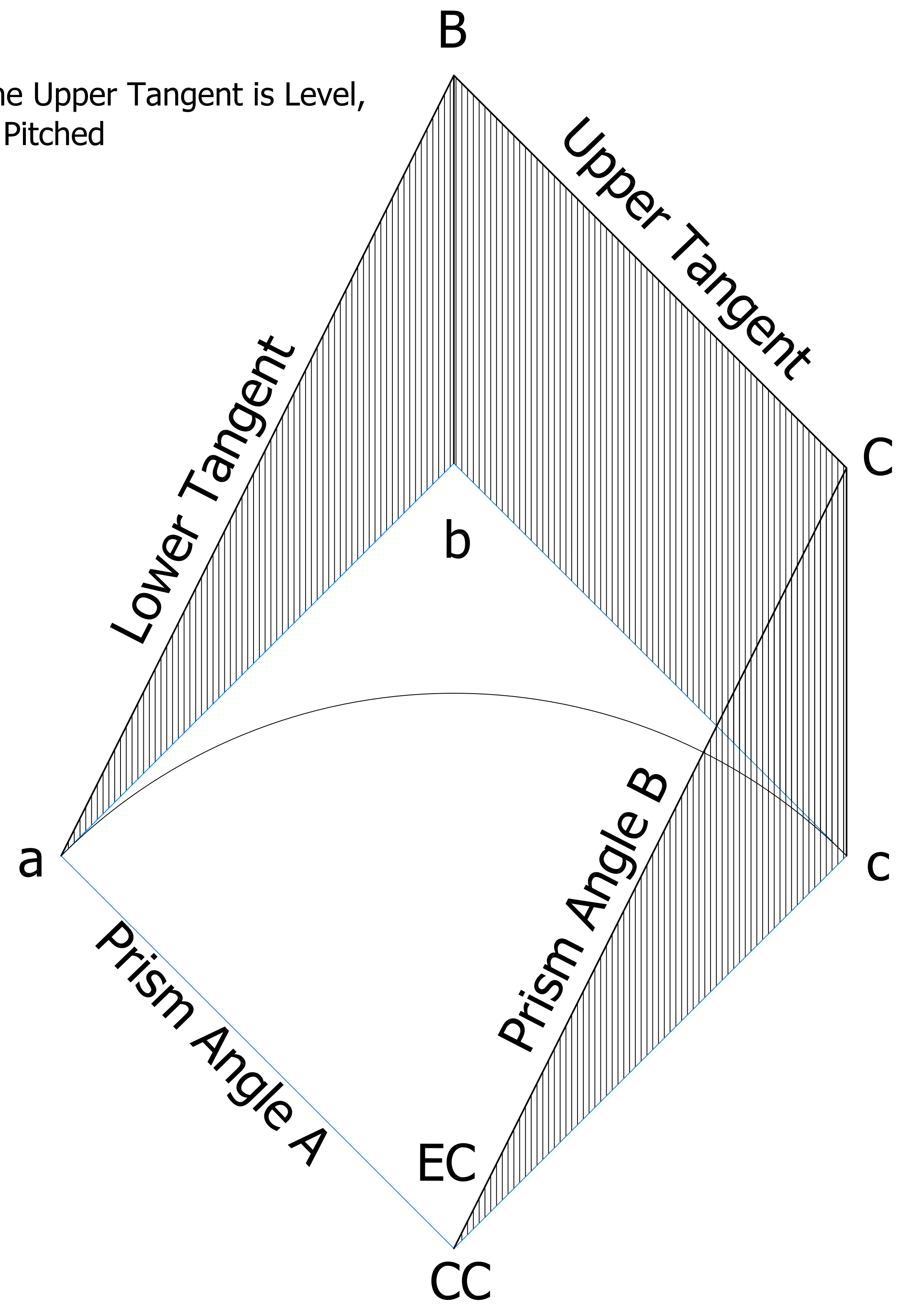
Plan View

Prism Angle A

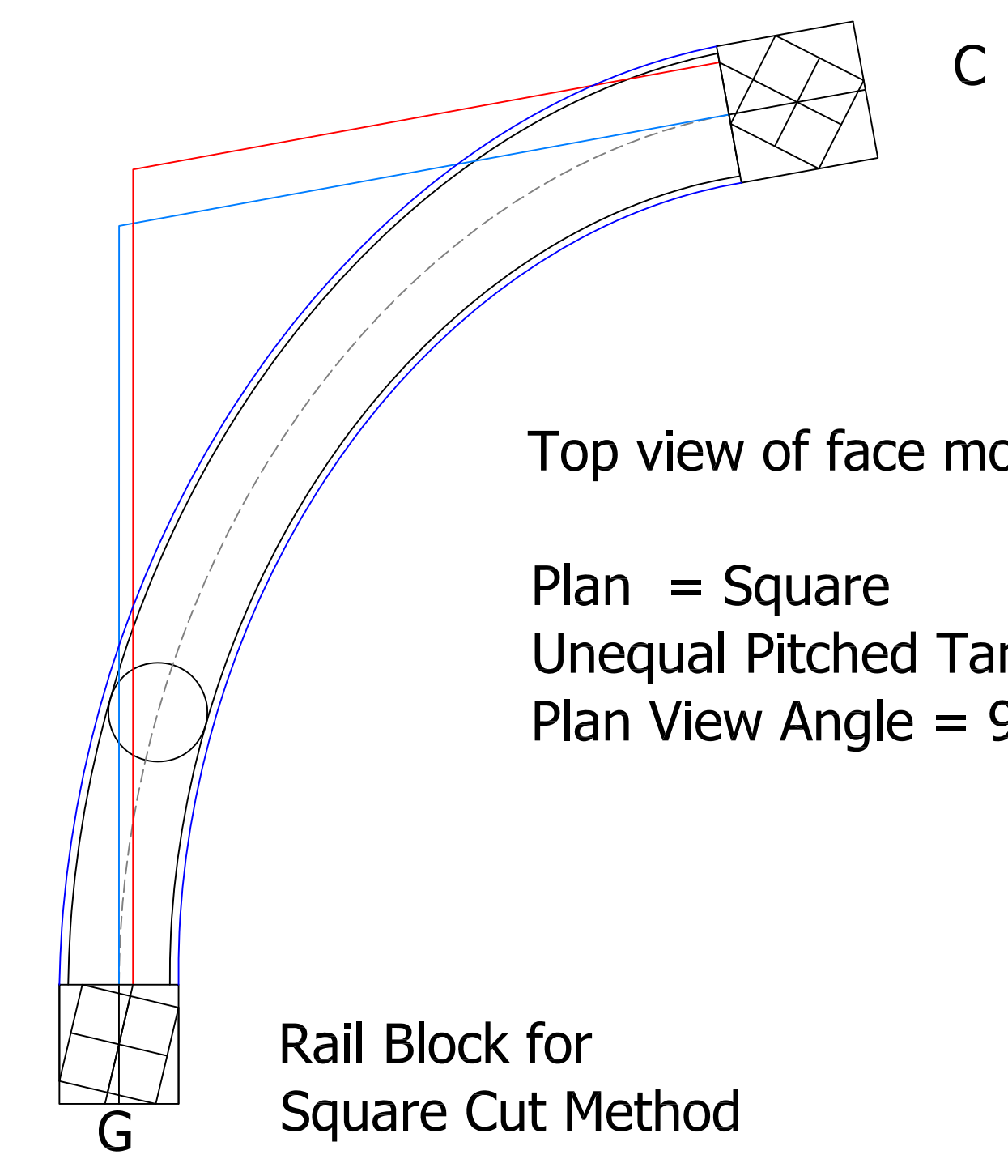
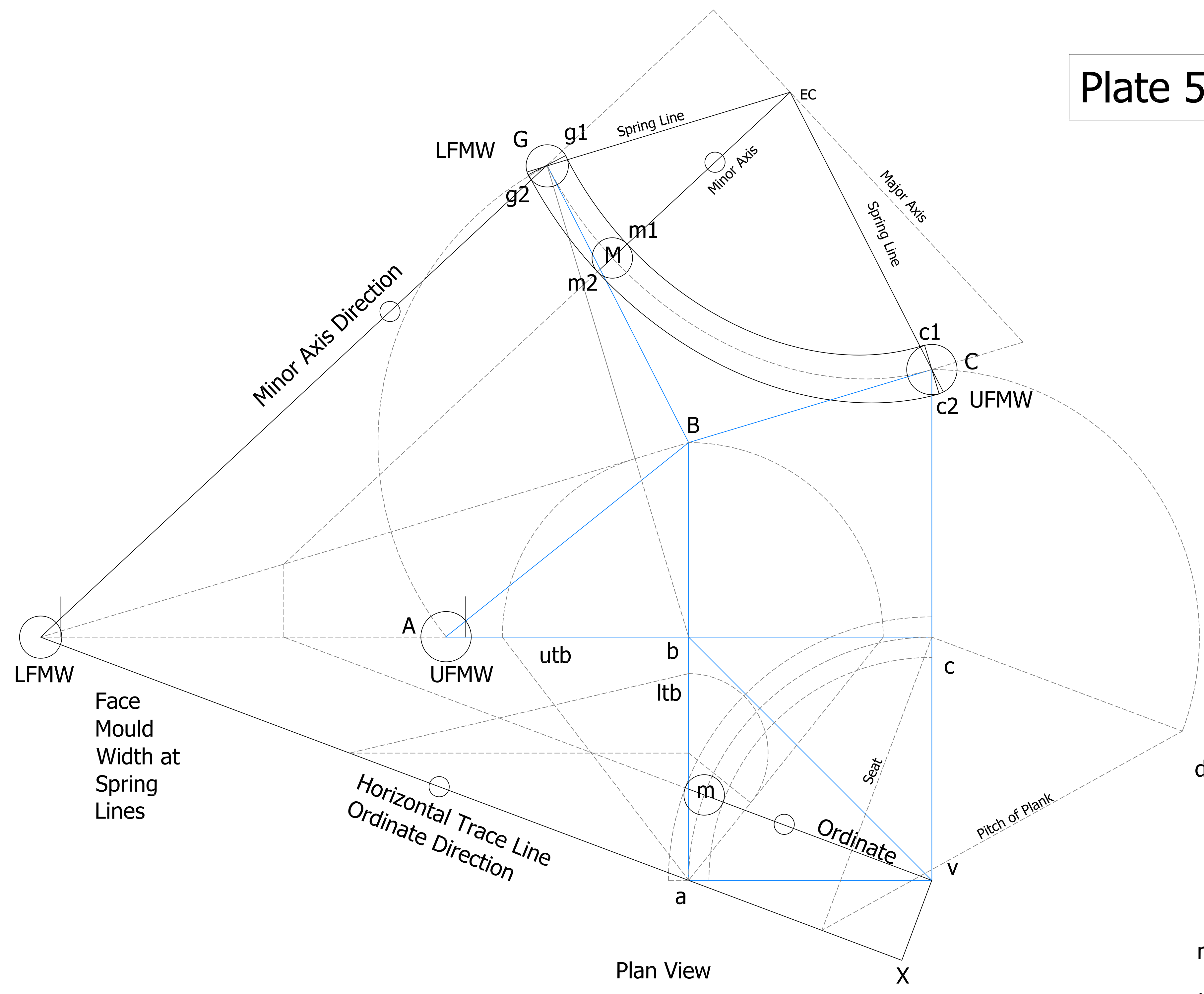
Prism Angle B

Height of Prism at EC Spring Lines  
is developed from pitch of plank

Isometric View of Prism



**Plate 5** Quarter-Circle Plan with Short Upper Pitched Tangent



Tangents in Original Position  
 Tangents shifted into Second Position

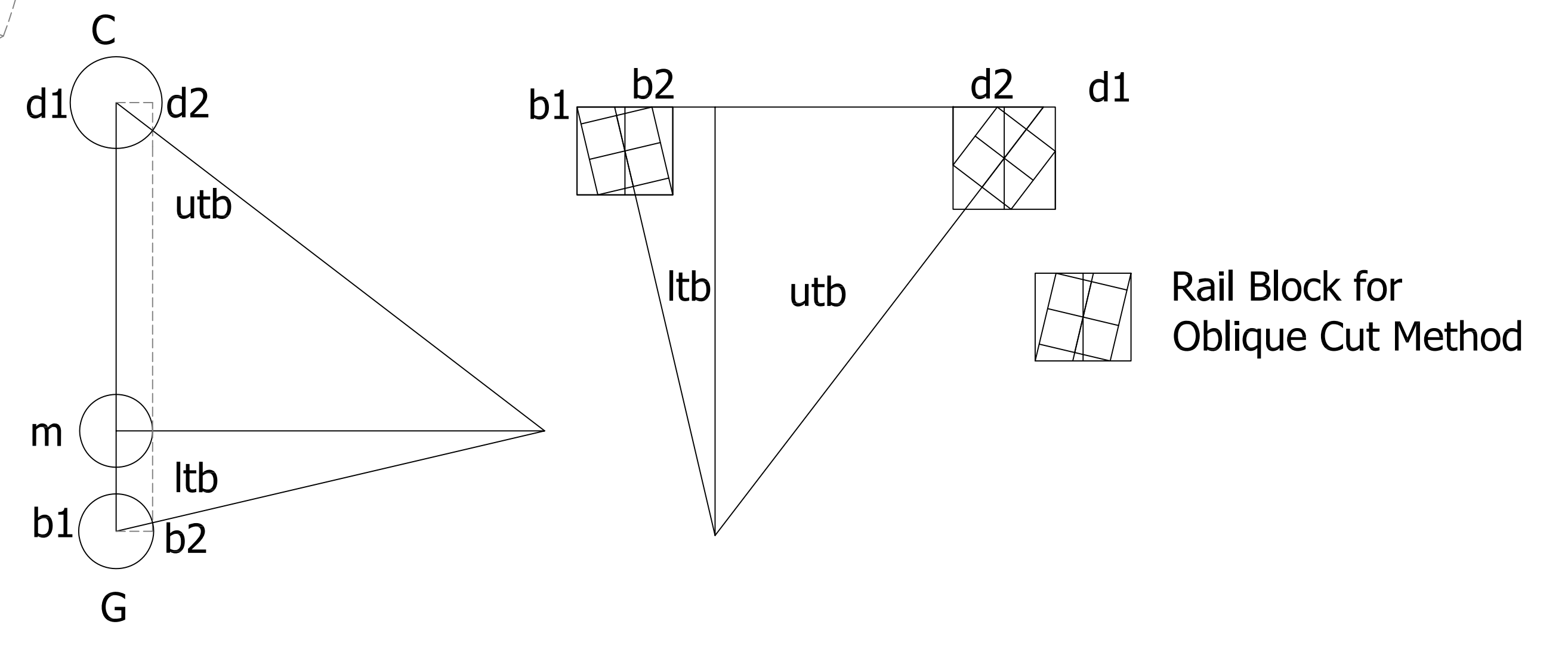
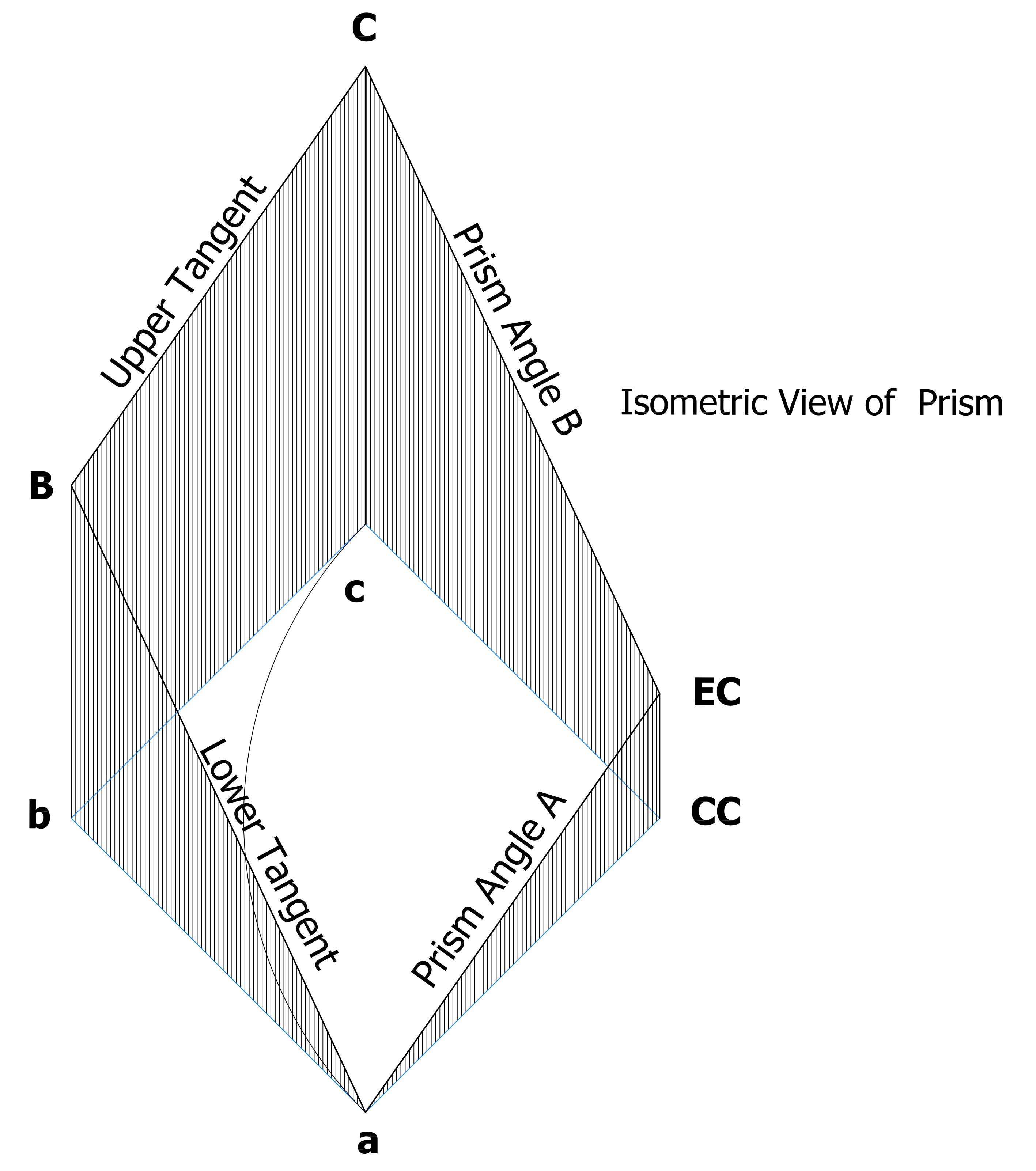
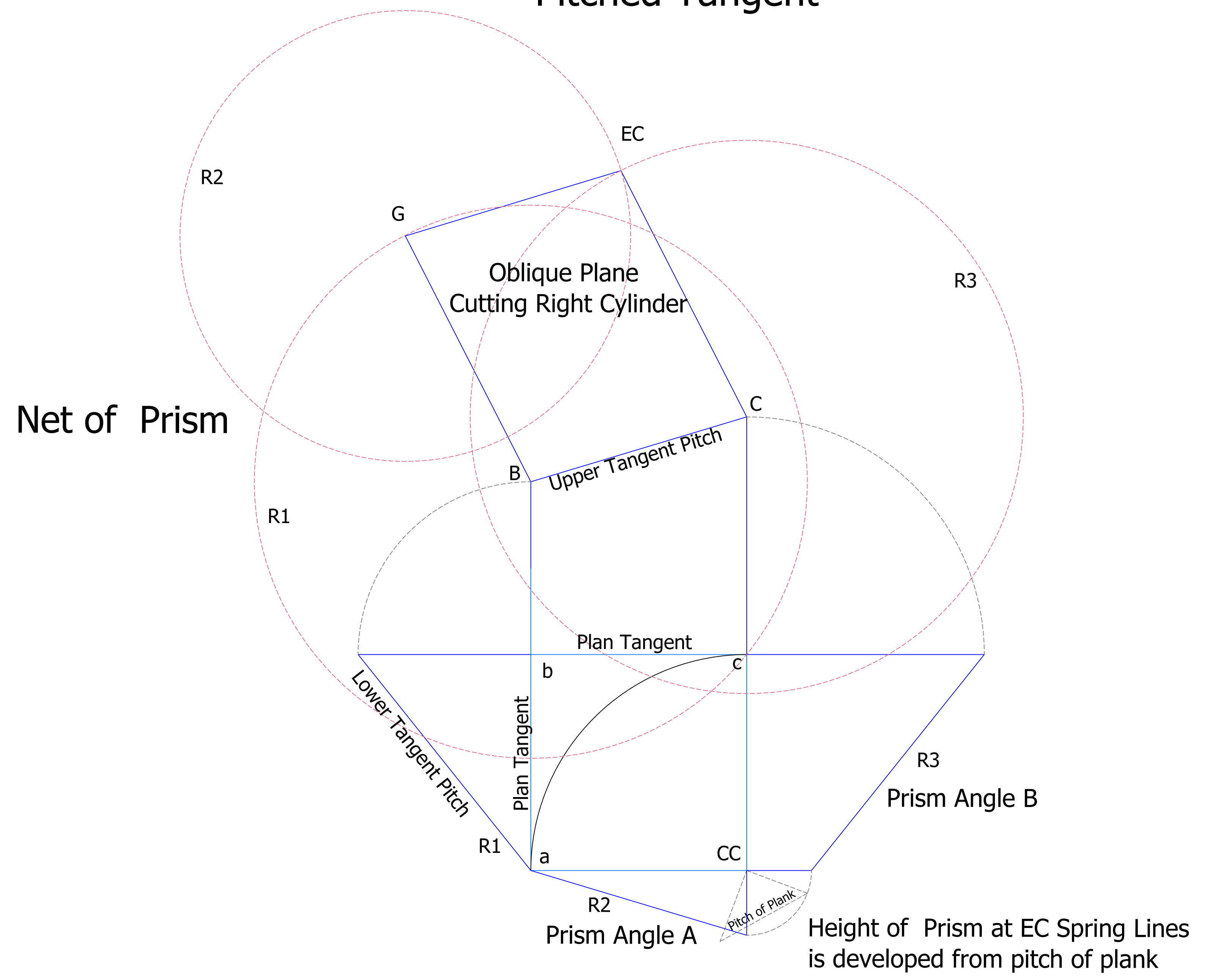
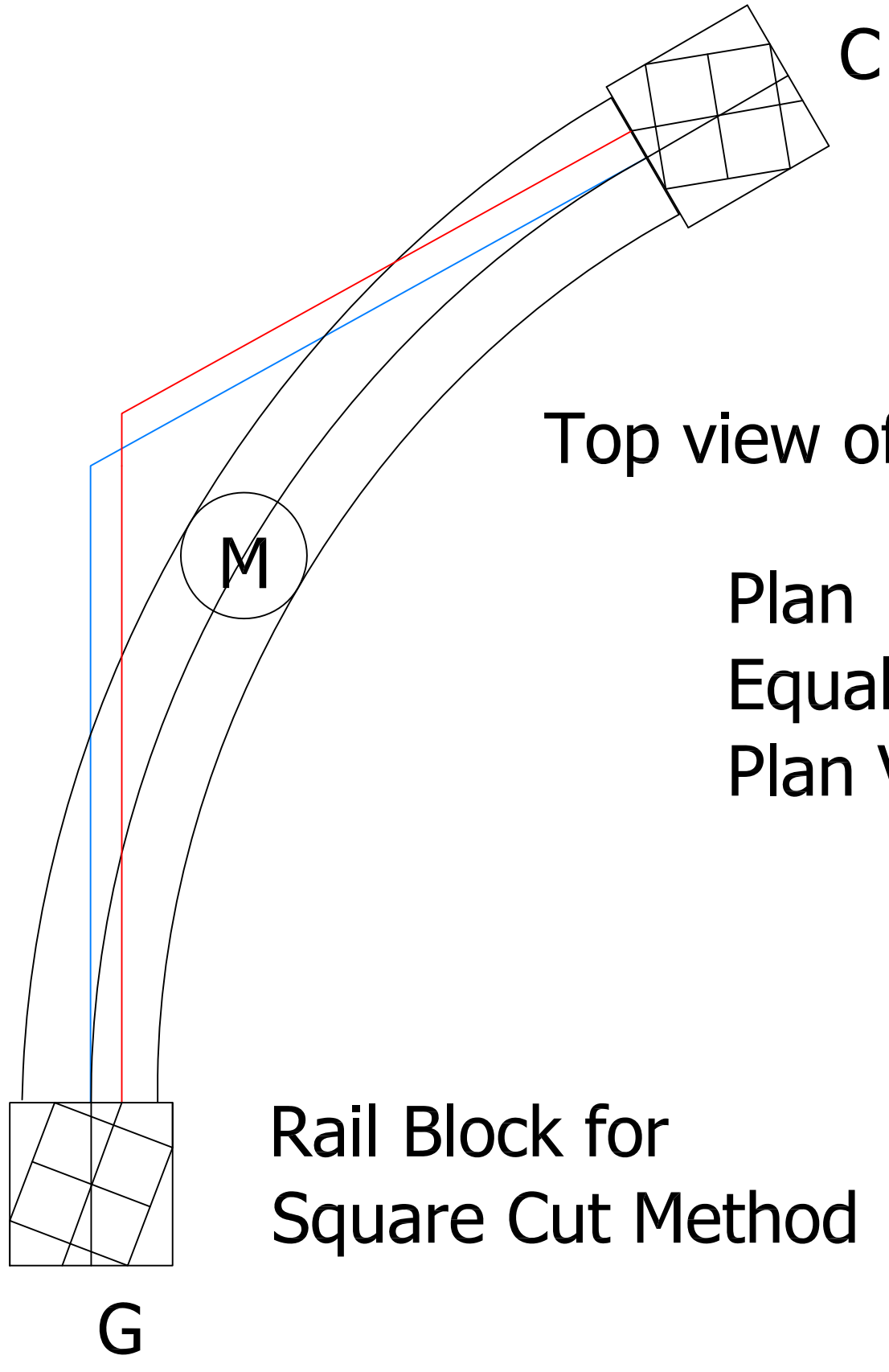
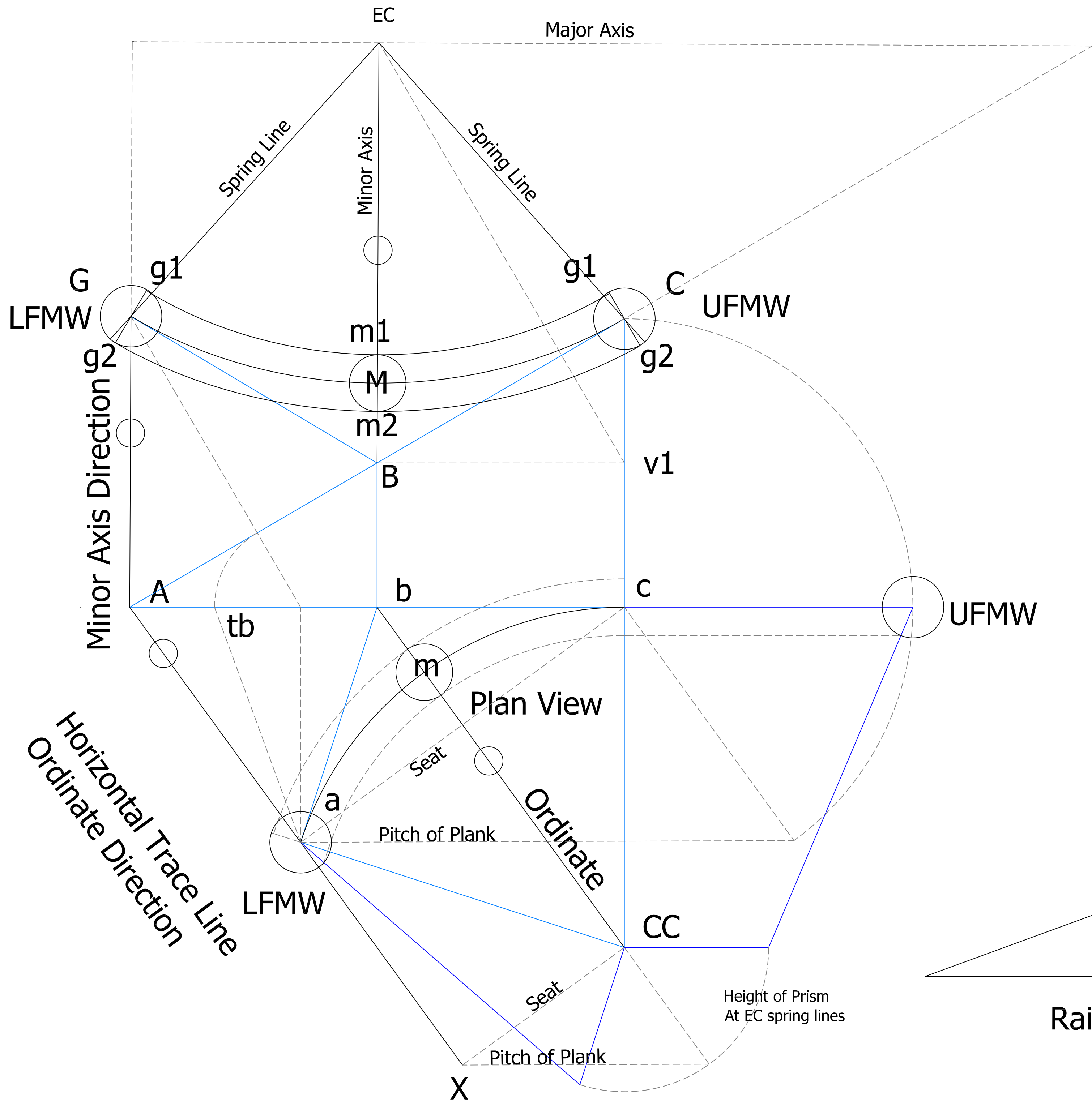


Plate 5.1 Quarter-Circle Plan with Short Upper Pitched Tangent

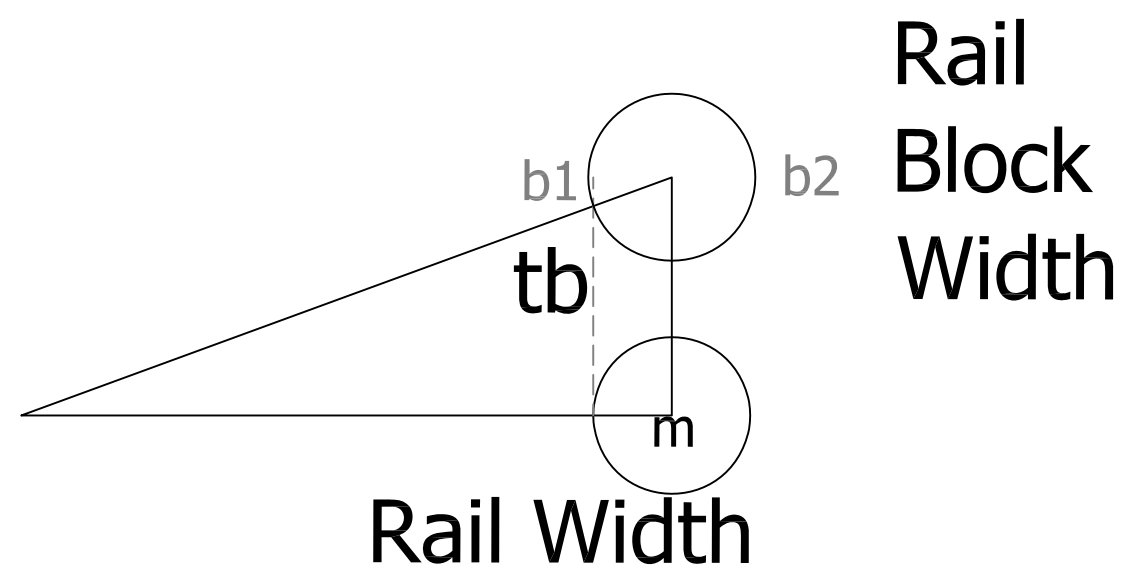


# Plate 6

## Obtuse Plan with Equally Pitched Tangents



Plan = Obtuse  
 Equal Pitched Tangents  
 Plan View Angle = 108.00



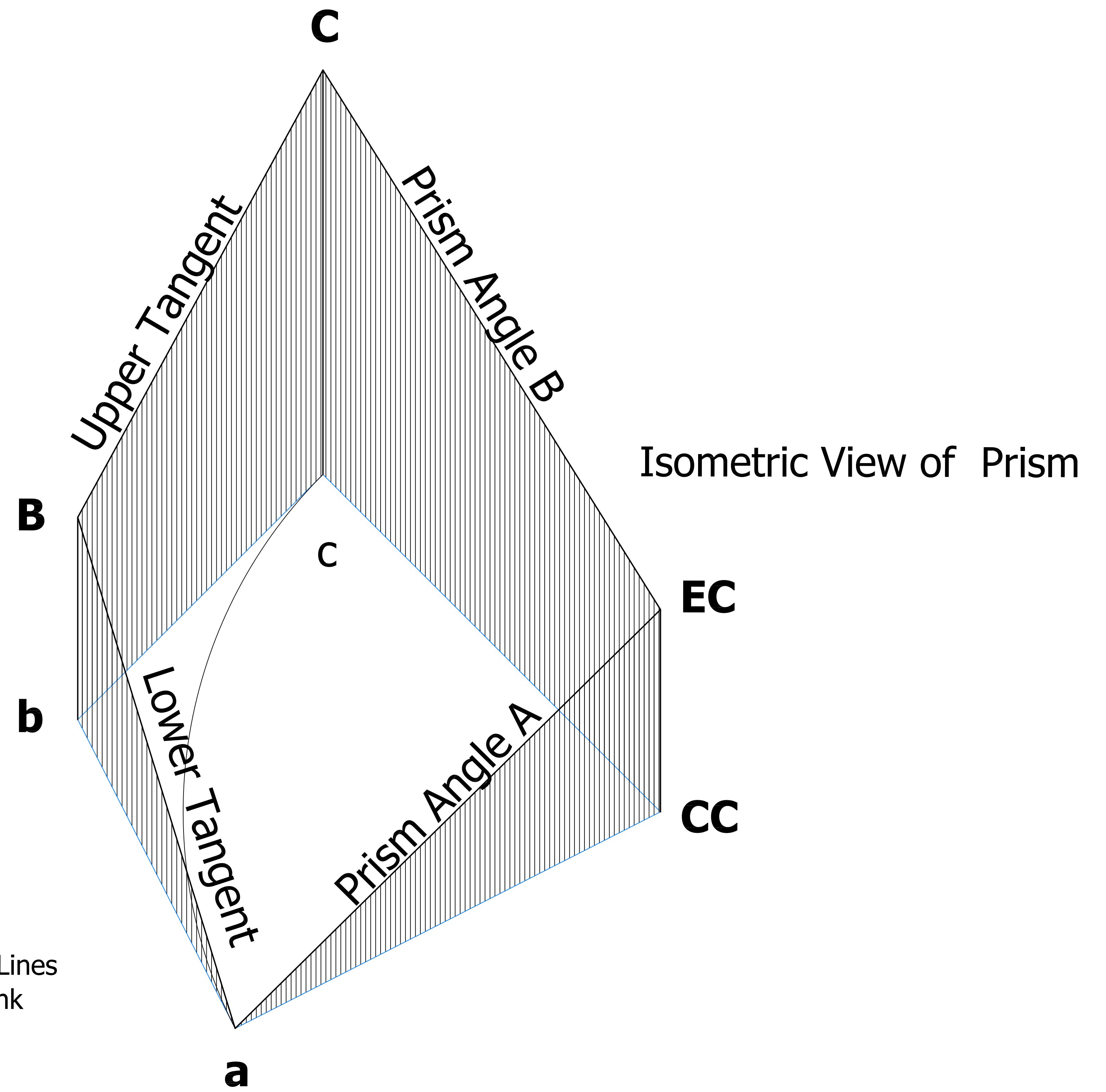
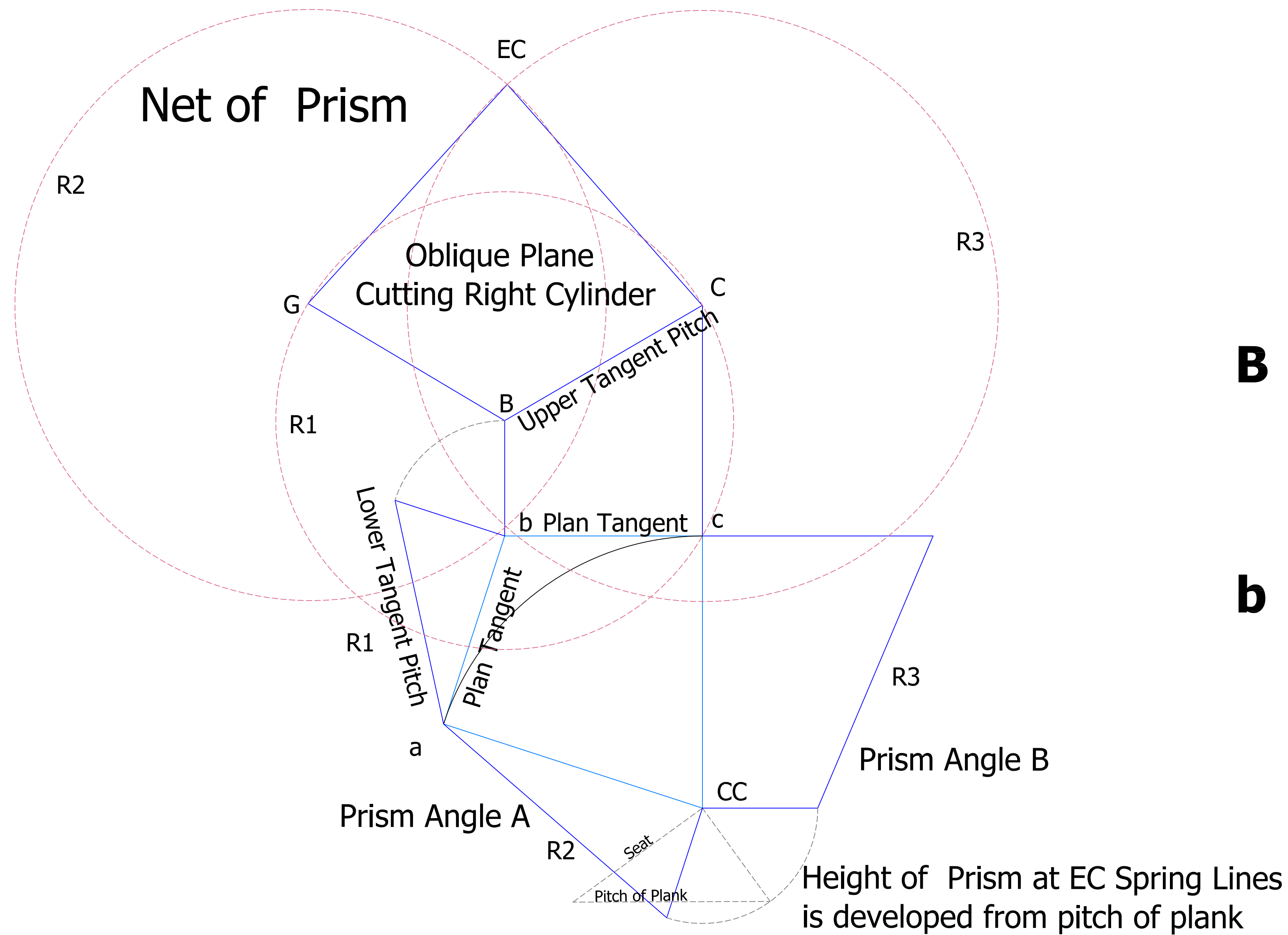
Tangents in Original Position

Tangents shifted into Second Position



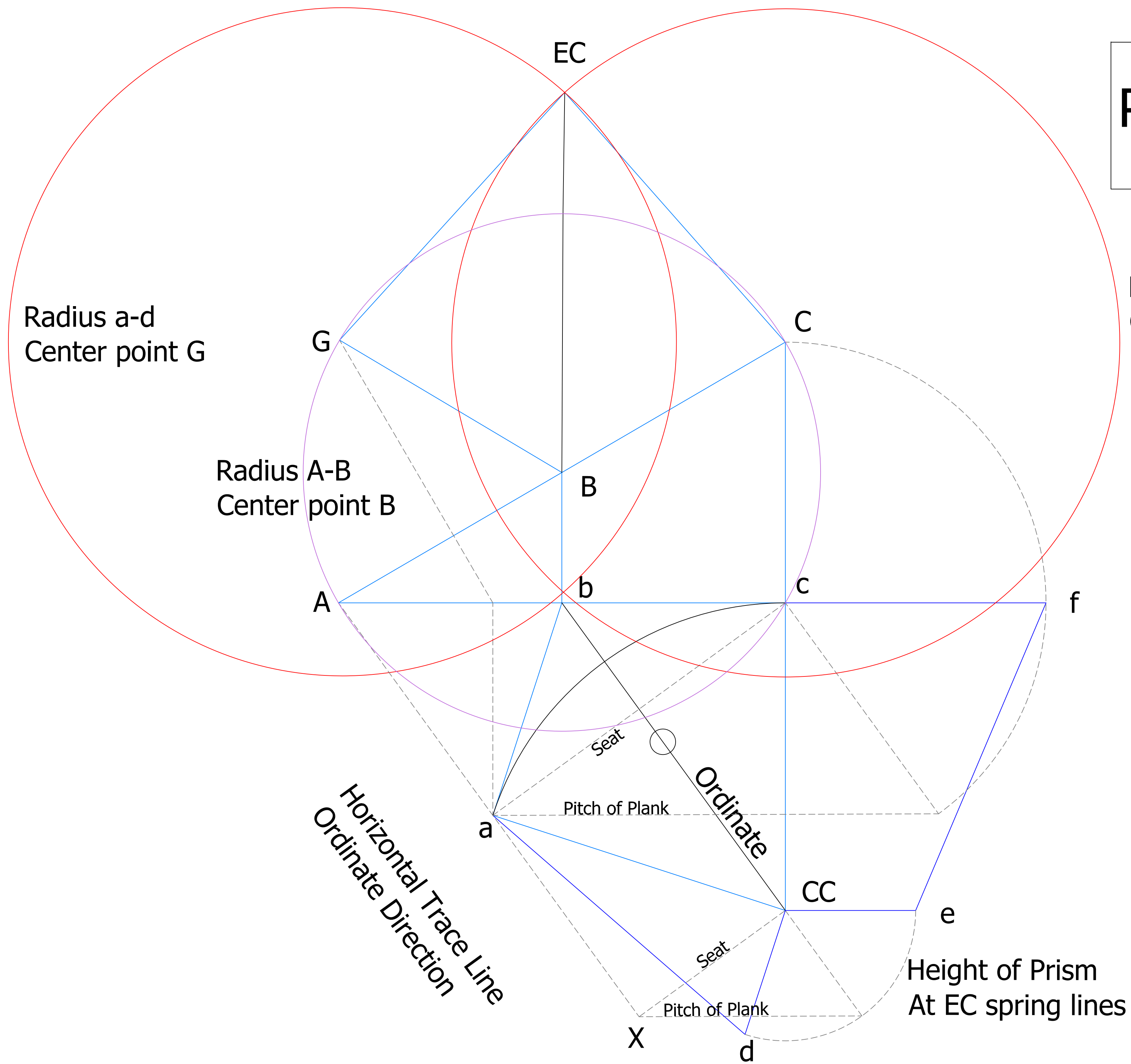
**Plate 6.1**

Obtuse Plan with Equally Pitched Tangents  
Locating ellipse center point and drawing  
the oblique plane



**Plate 6.1**

Obtuse Plan with Equally Pitched Tangents  
 Locating ellipse center point and drawing  
 the oblique plane

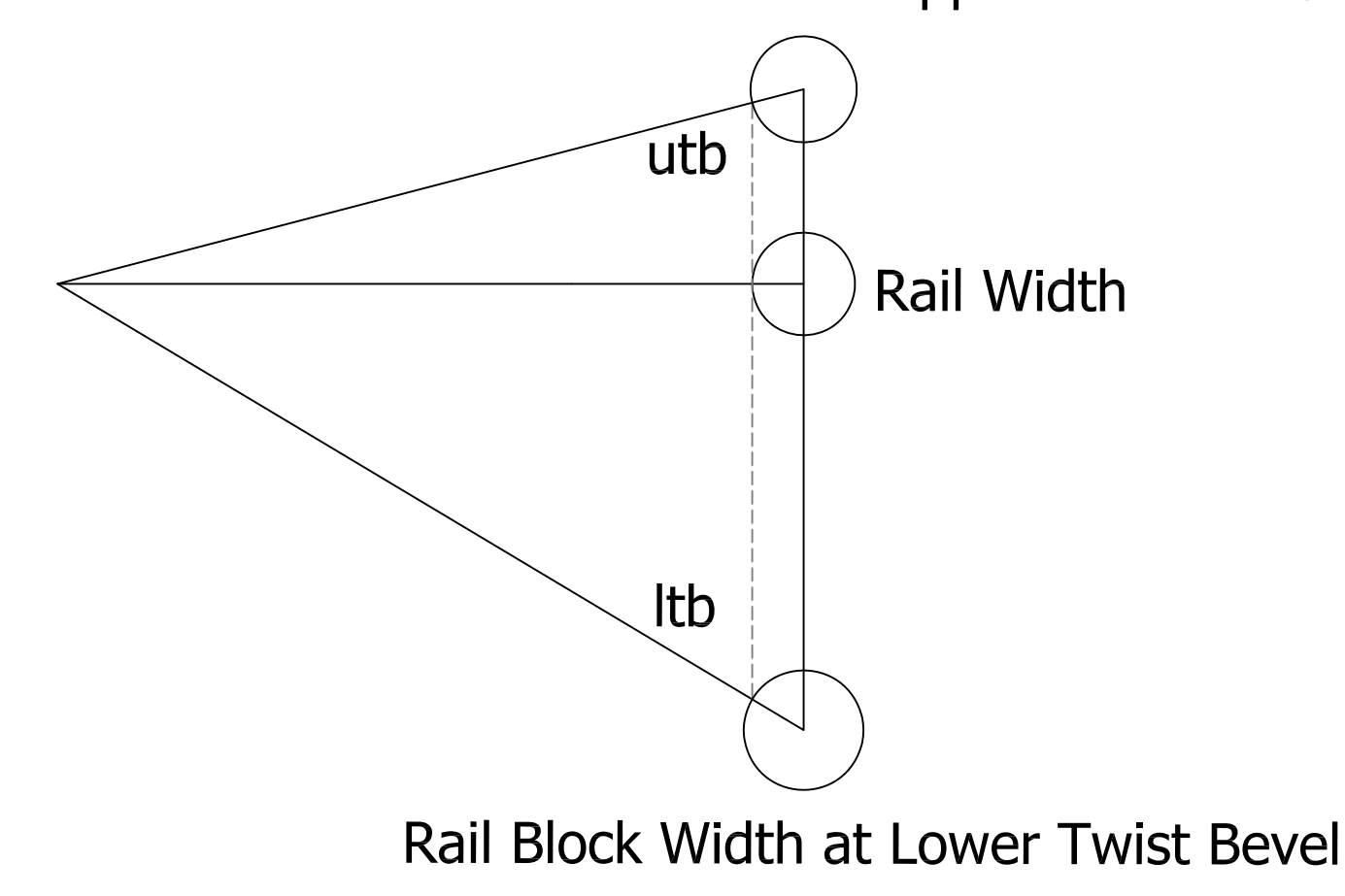
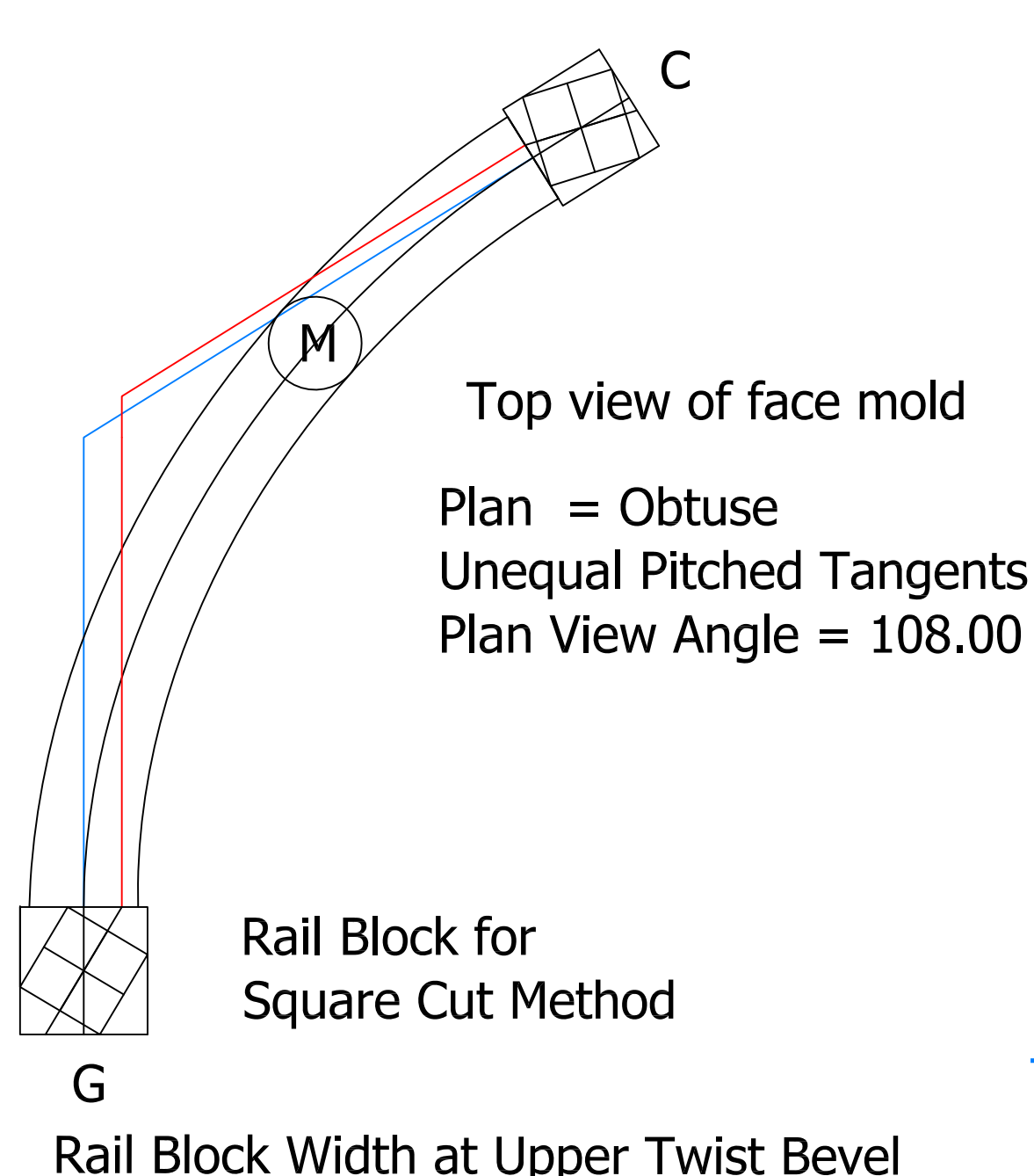
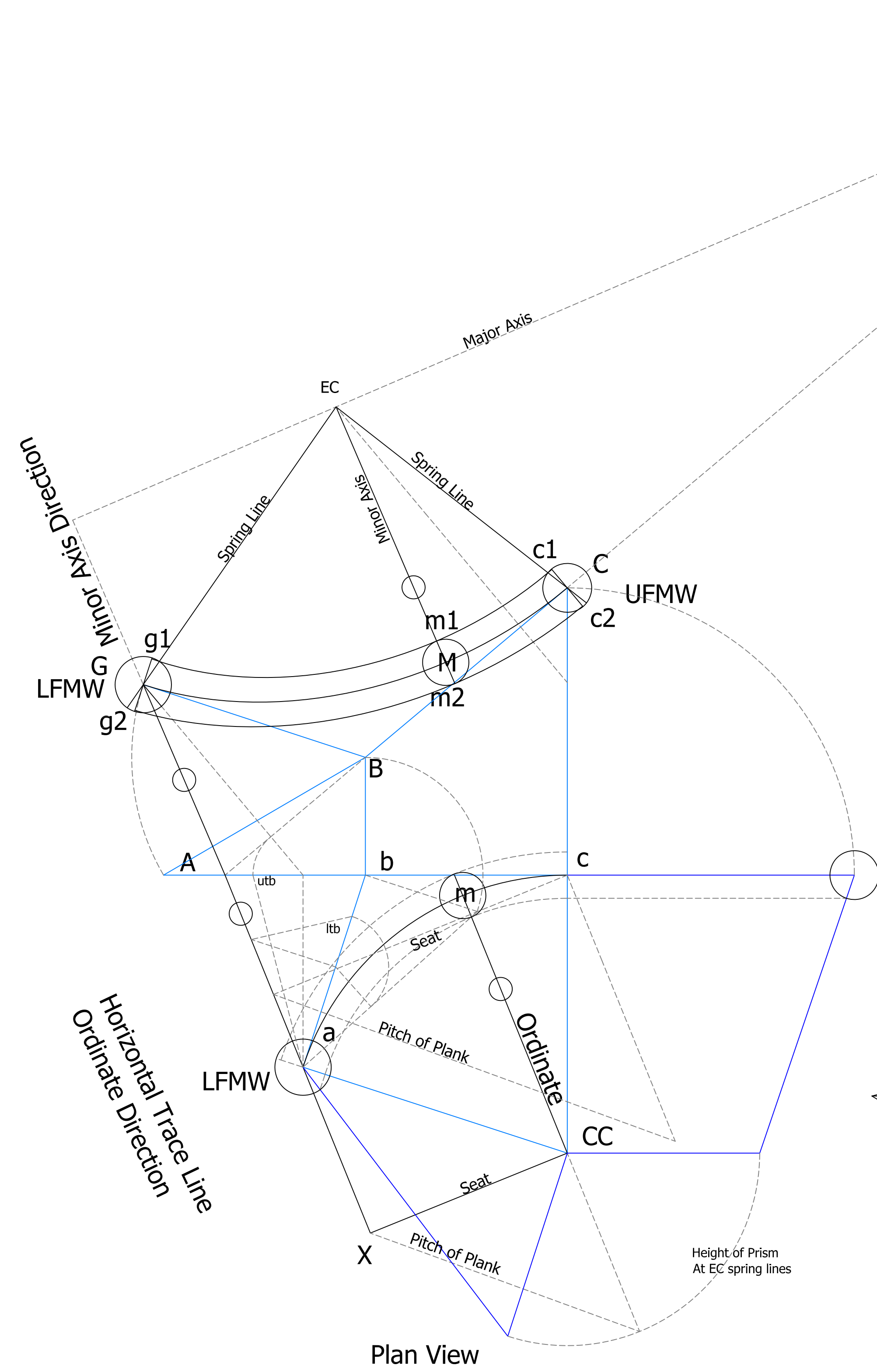


Radius e-f  
 Center point C

Plan = Obtuse  
 Equal Pitched Tangents  
 Plan View Angle = 108.00

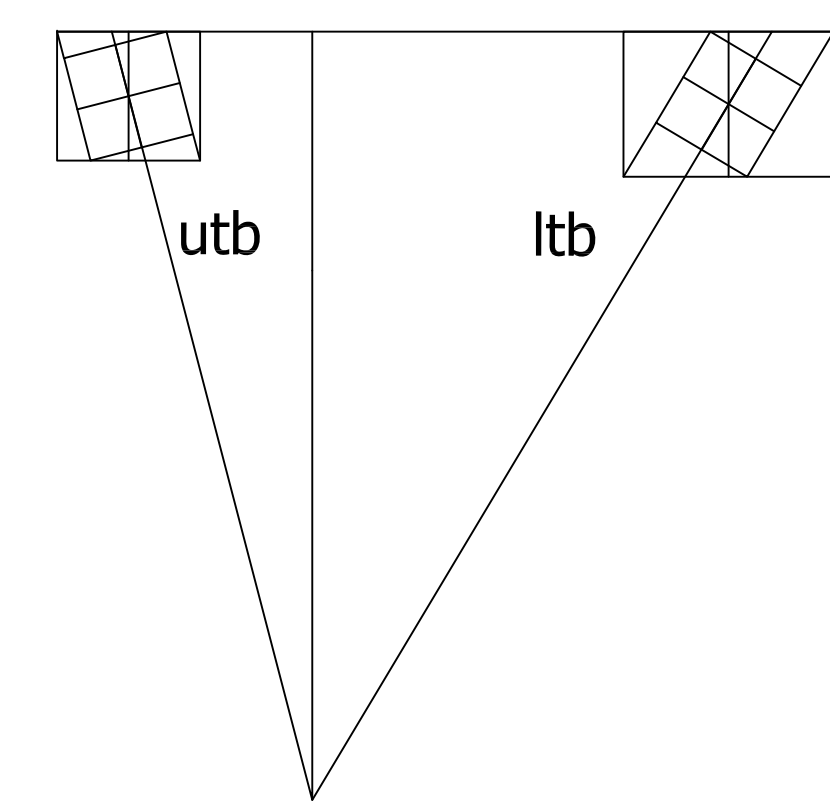
**Plate 7**

Obtuse Plan with Short Lower Pitched Tangent



Tangents in Original Position

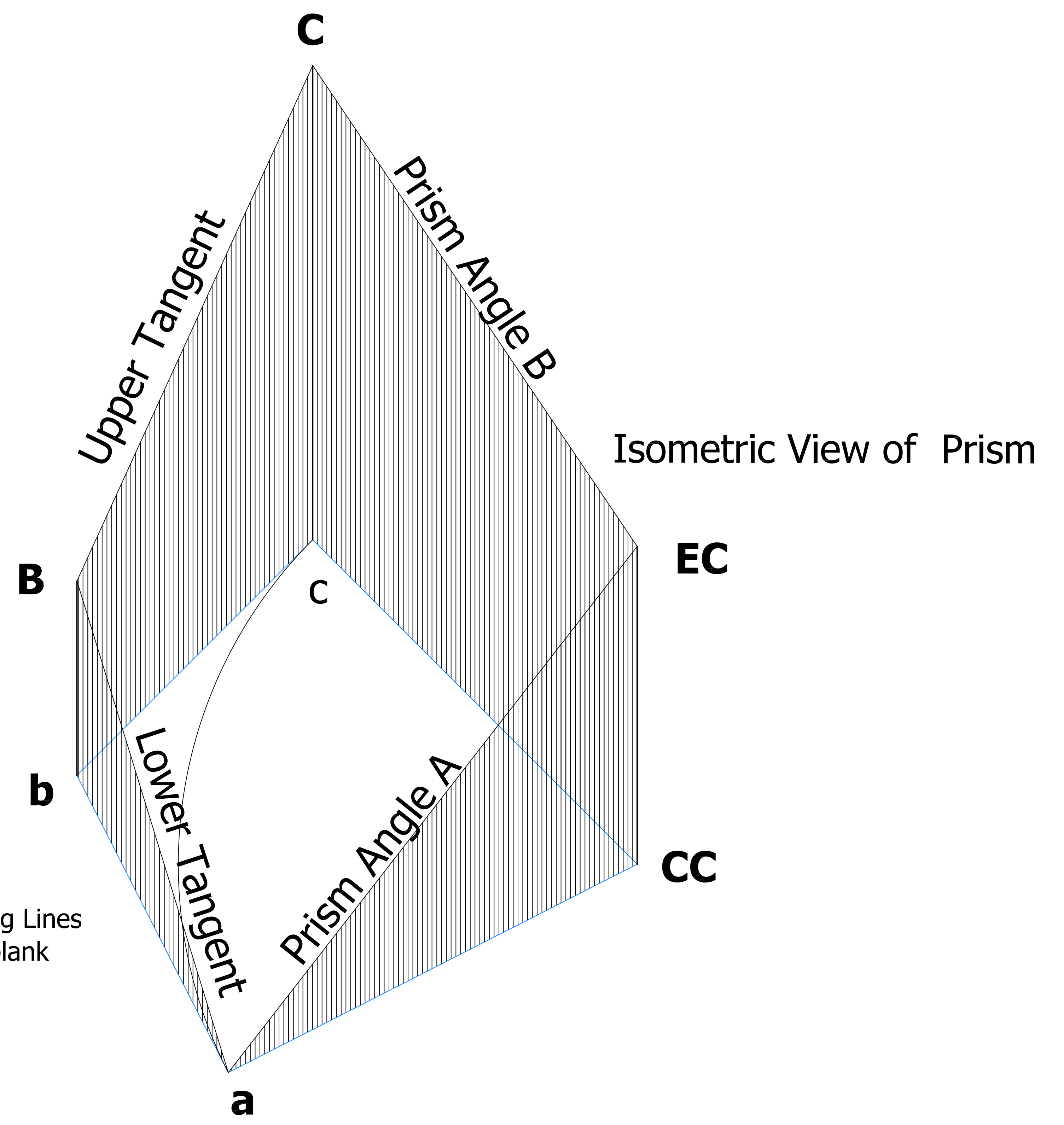
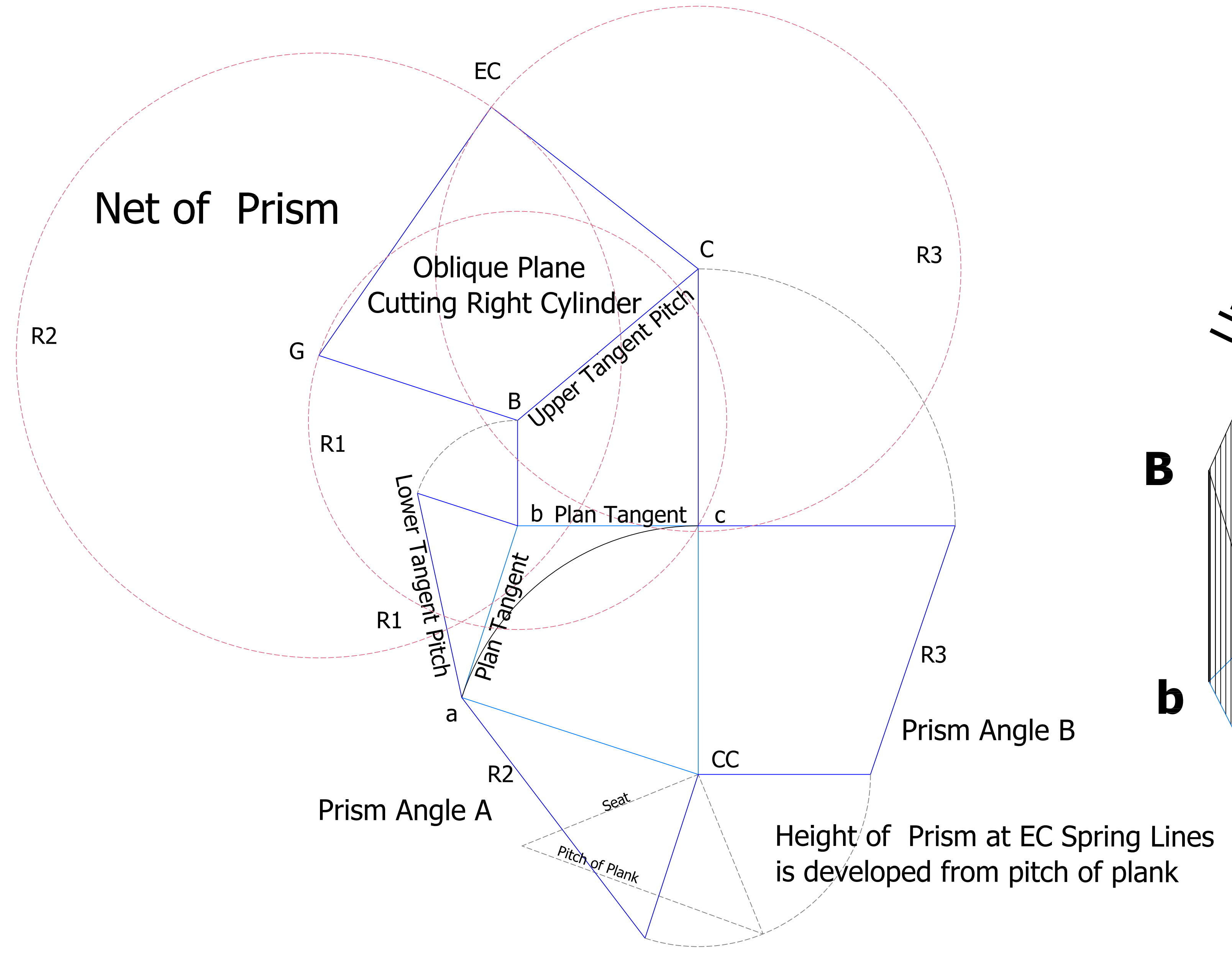
Tangents shifted into Second Position

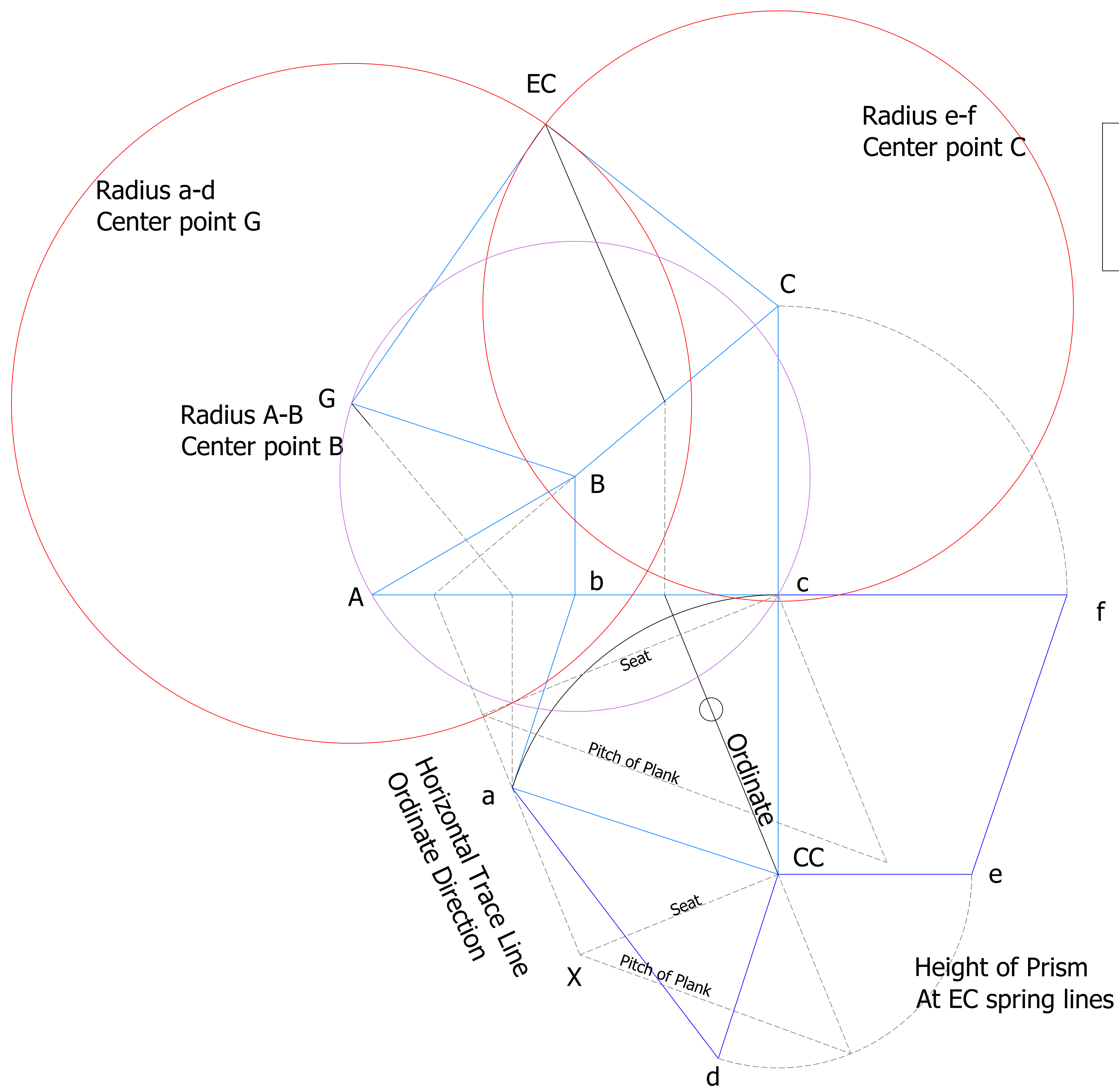




**Plate 7.1**

Obtuse Plan with Short Lower Pitched Tangent  
 Locating ellipse center point and drawing  
 the oblique plane





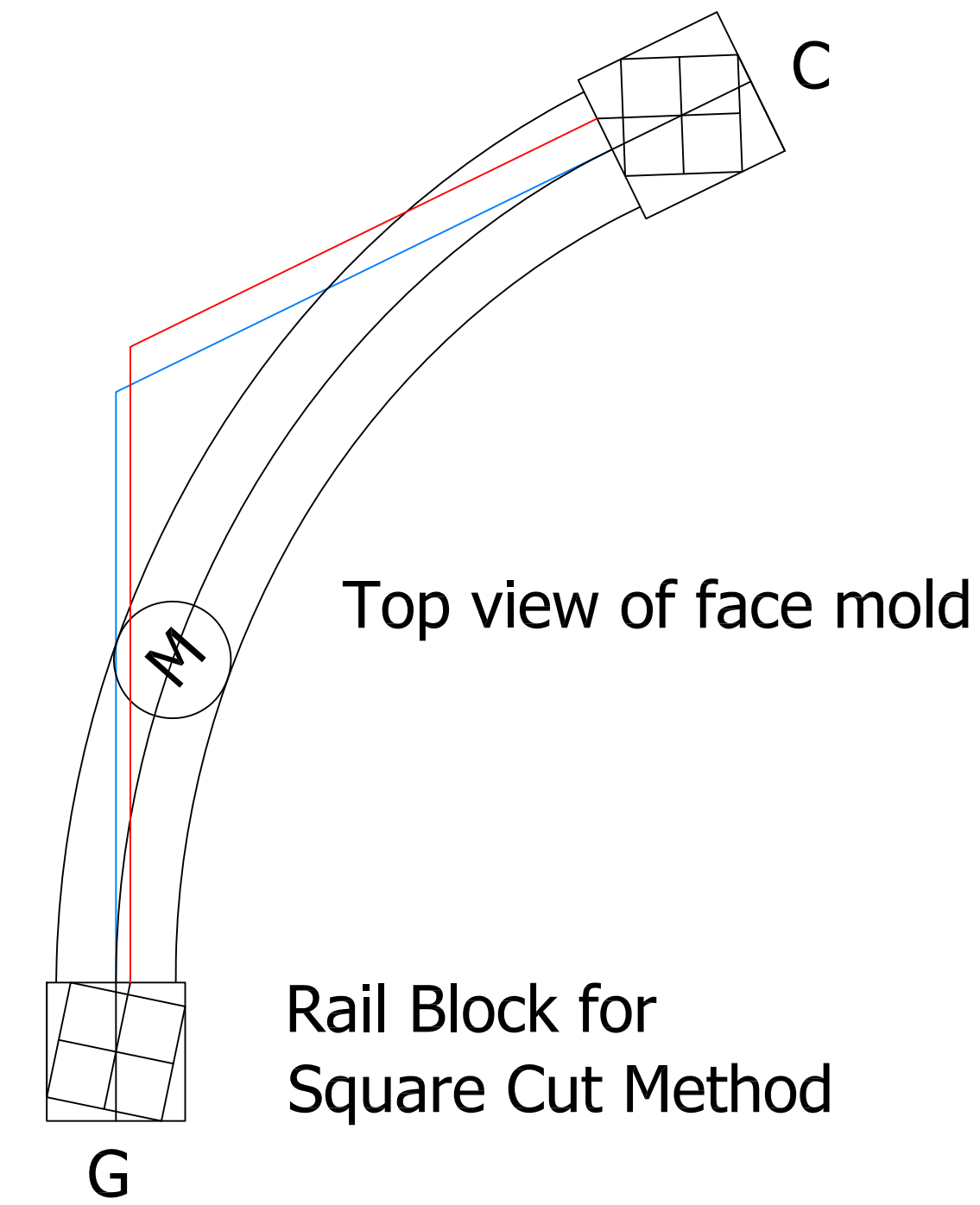
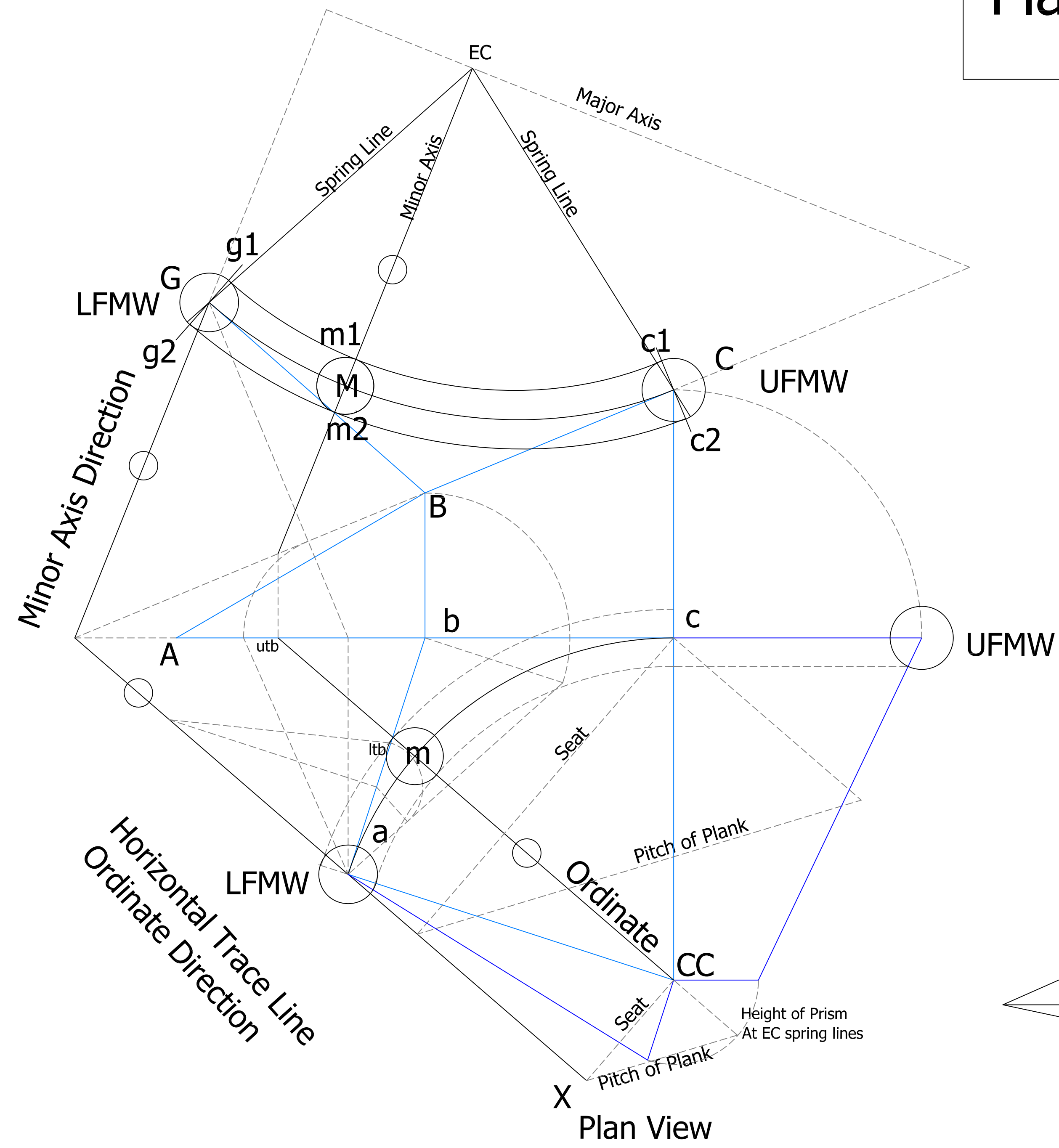
**Plate 7.1**

Obtuse Plan with Short Lower Pitched Tangent  
 Locating ellipse center point and drawing  
 the oblique plane

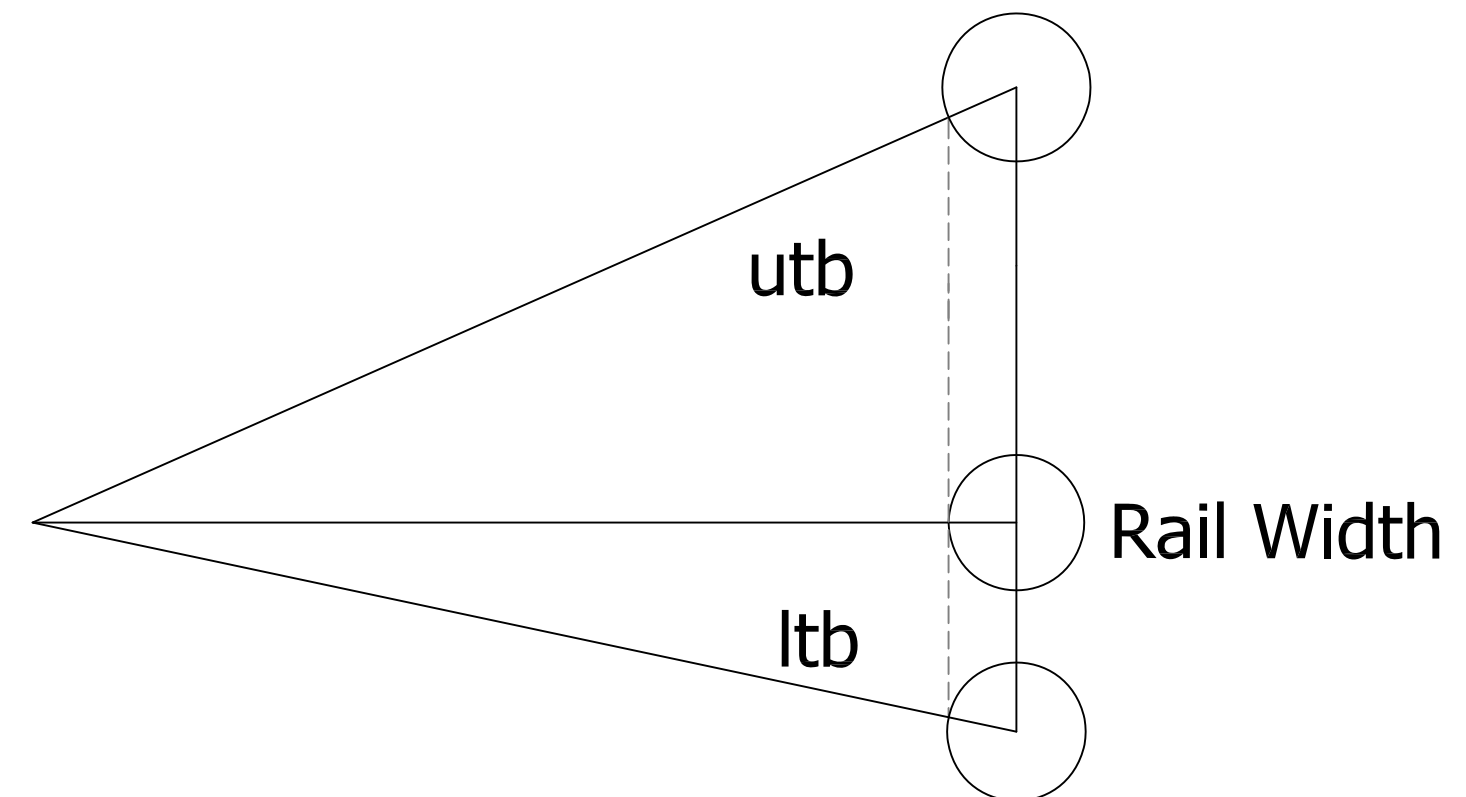
Plan = Obtuse  
 Unequal Pitched Tangents  
 Plan View Angle = 108.00

# Plate 8

## Obtuse Plan with Short Upper Pitched Tangent



Rail Block Width at Upper Twist Bevel

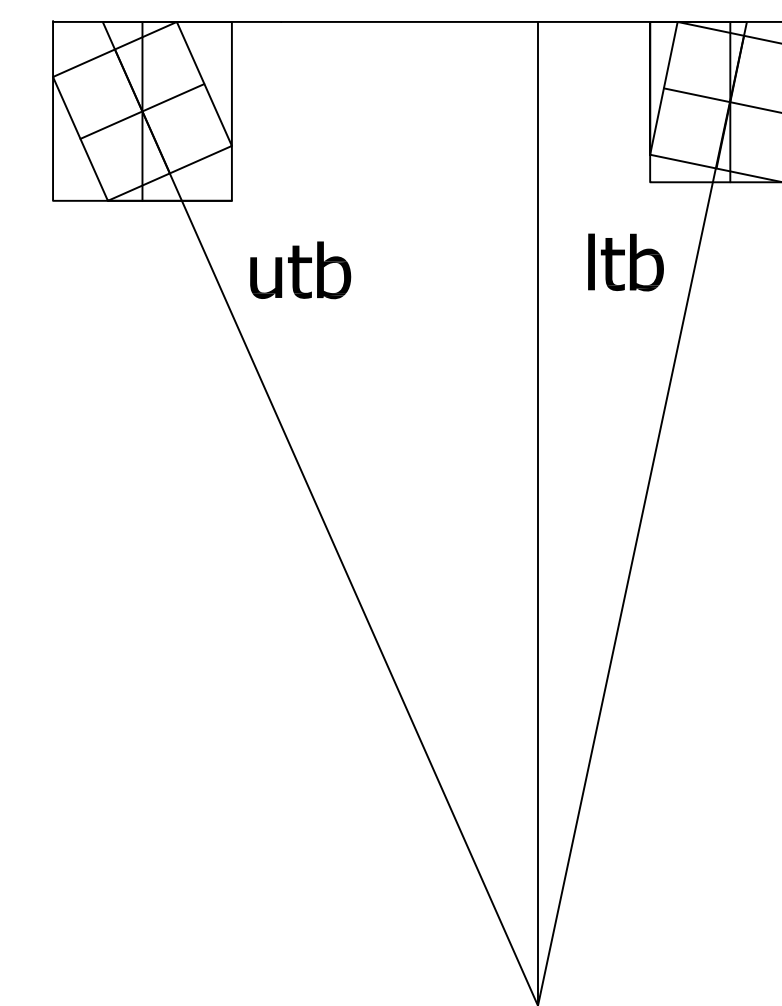


Rail Block Width at Lower Twist Bevel

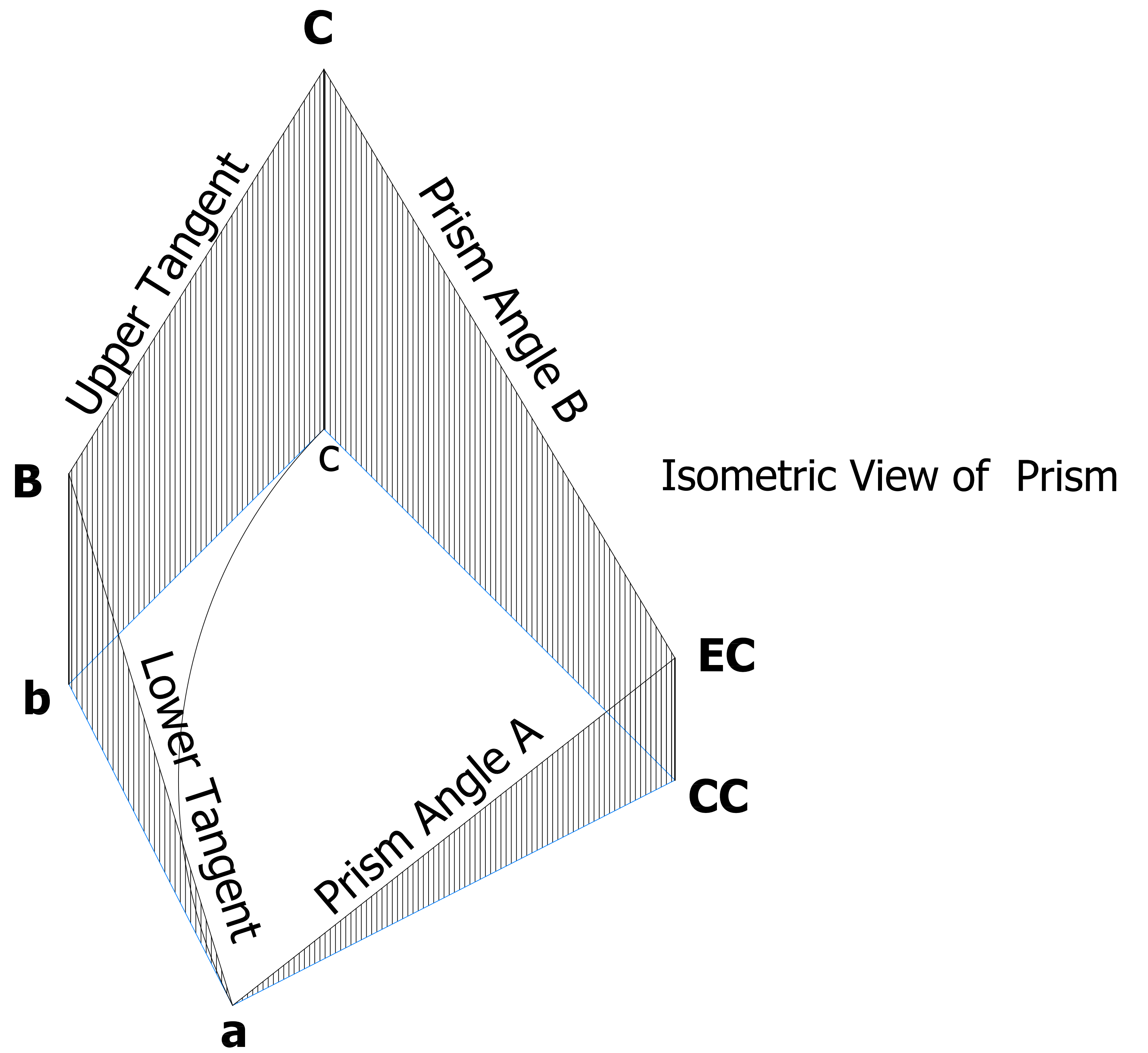
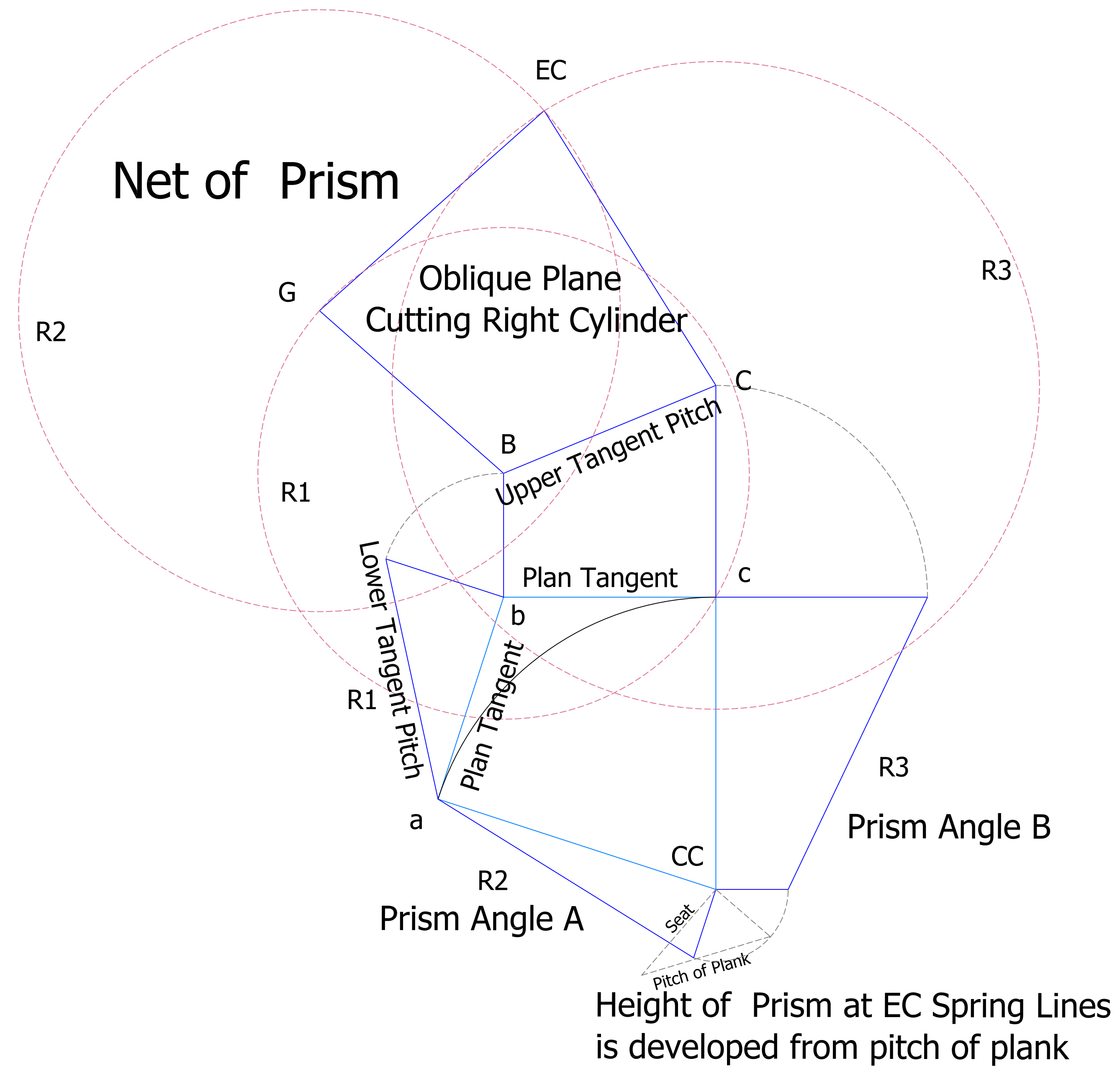
Plan = Obtuse  
 Unequal Pitched Tangents  
 Plan View Angle = 108.00

Tangents in Original Position

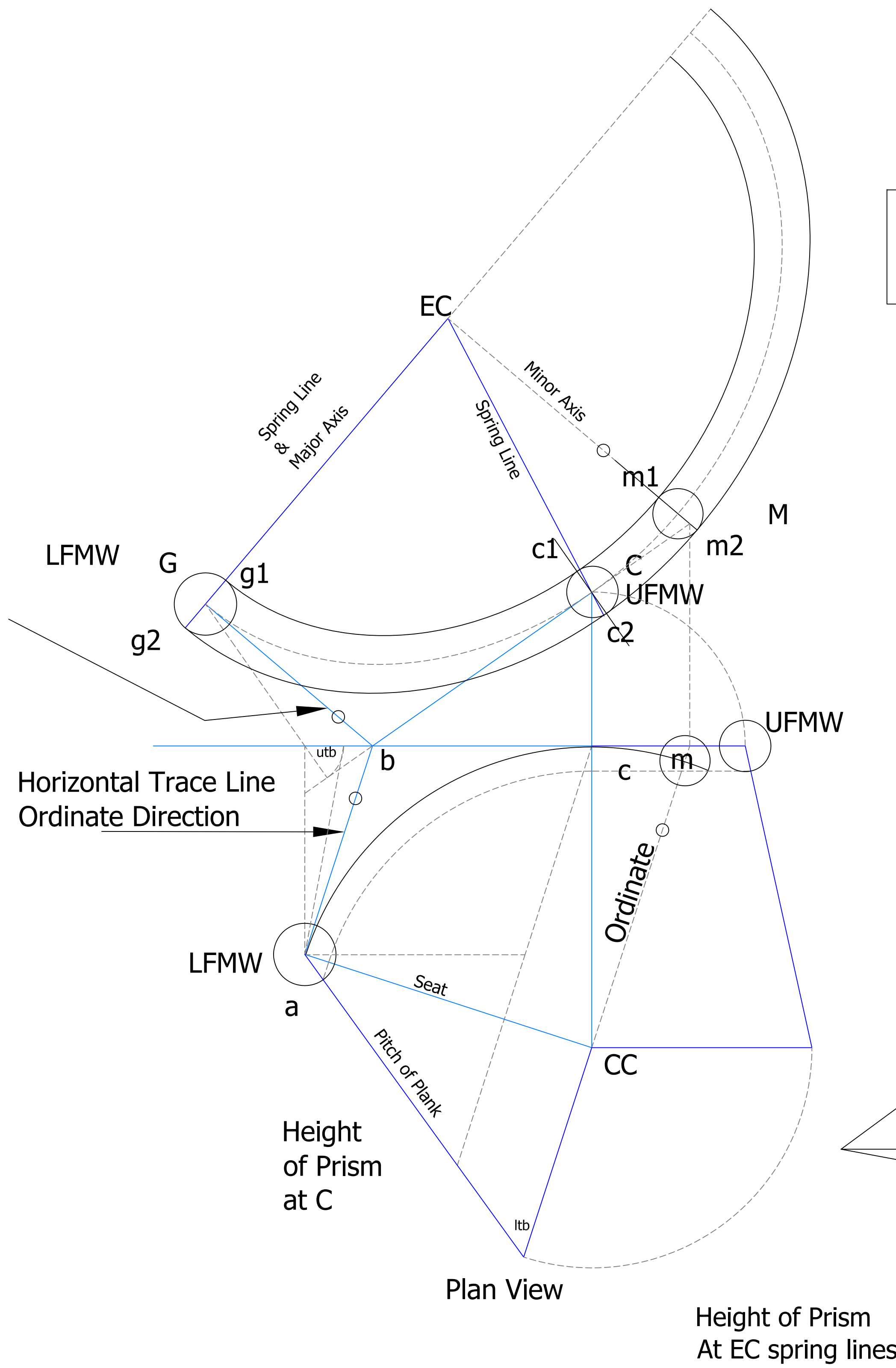
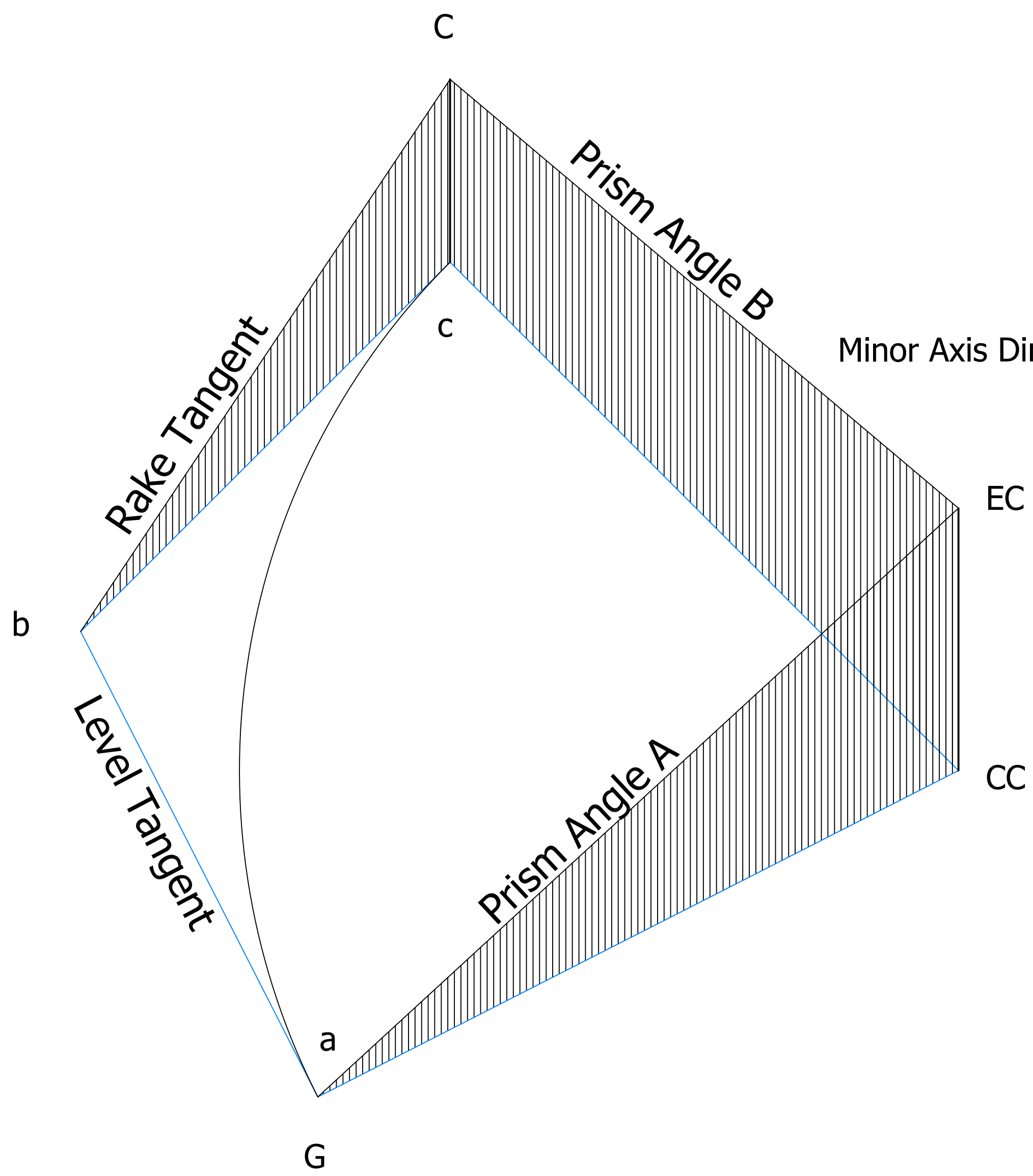
Tangents shifted into Second Position



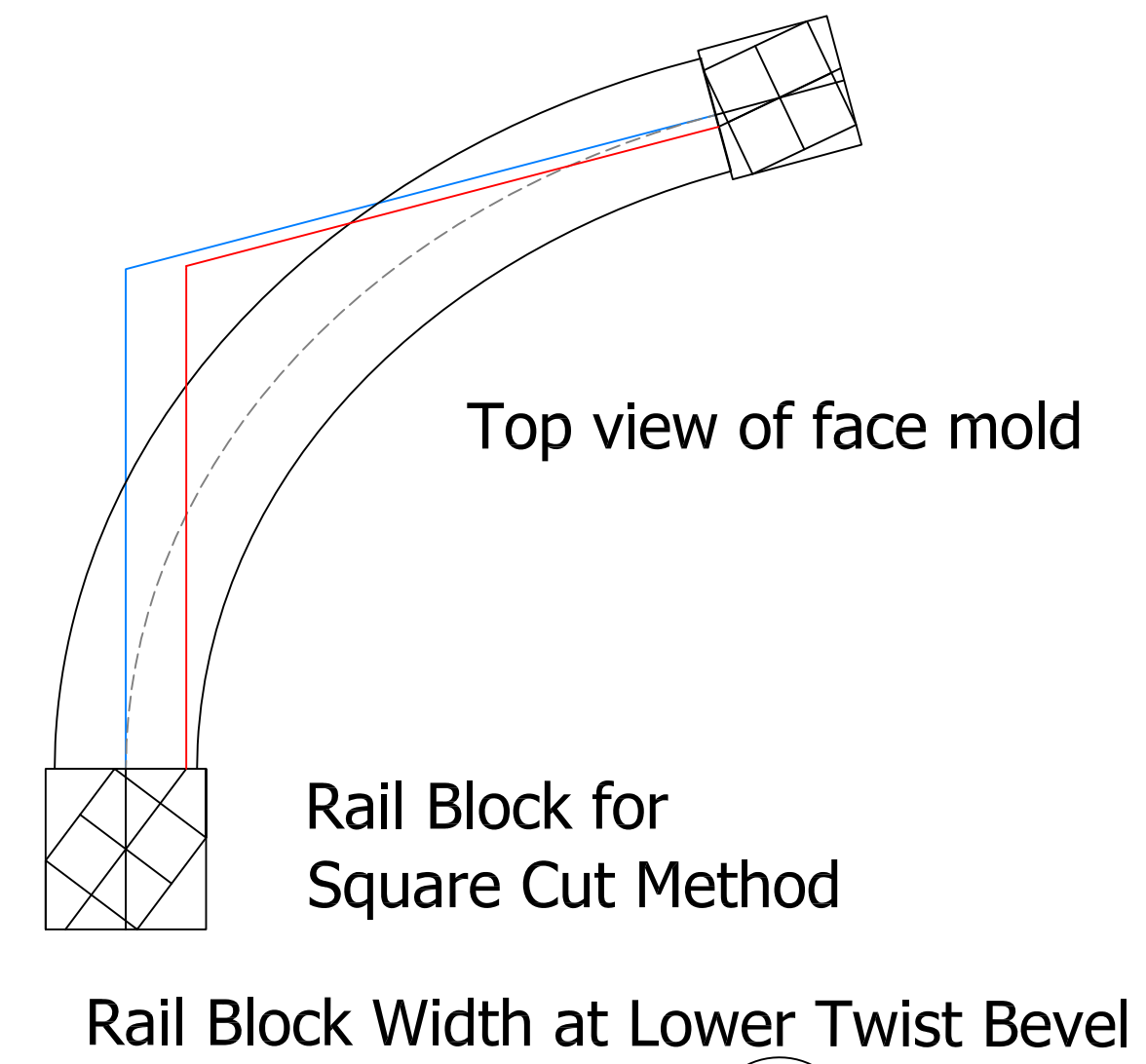
**Plate 8.1** Obtuse Plan with Short Upper Pitched Tangent



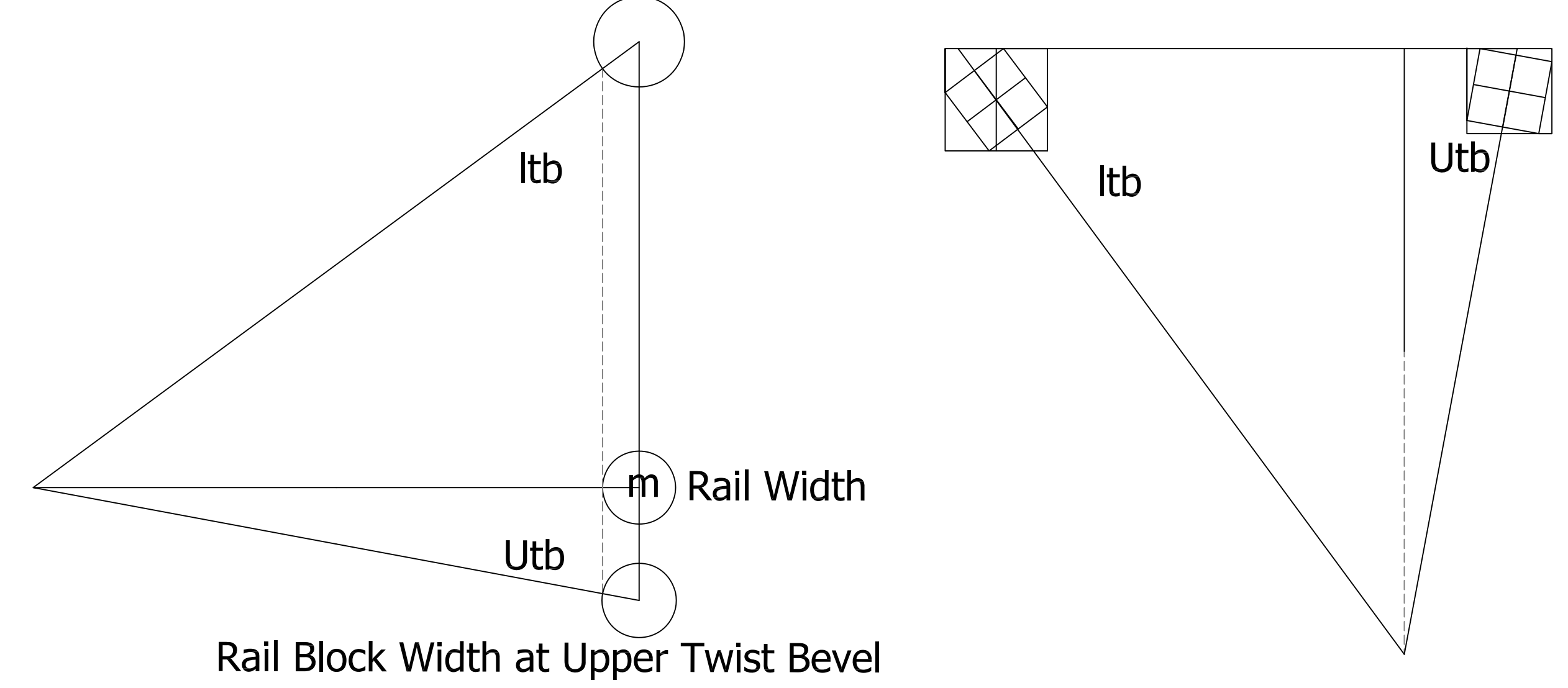
Isometric View of Prism

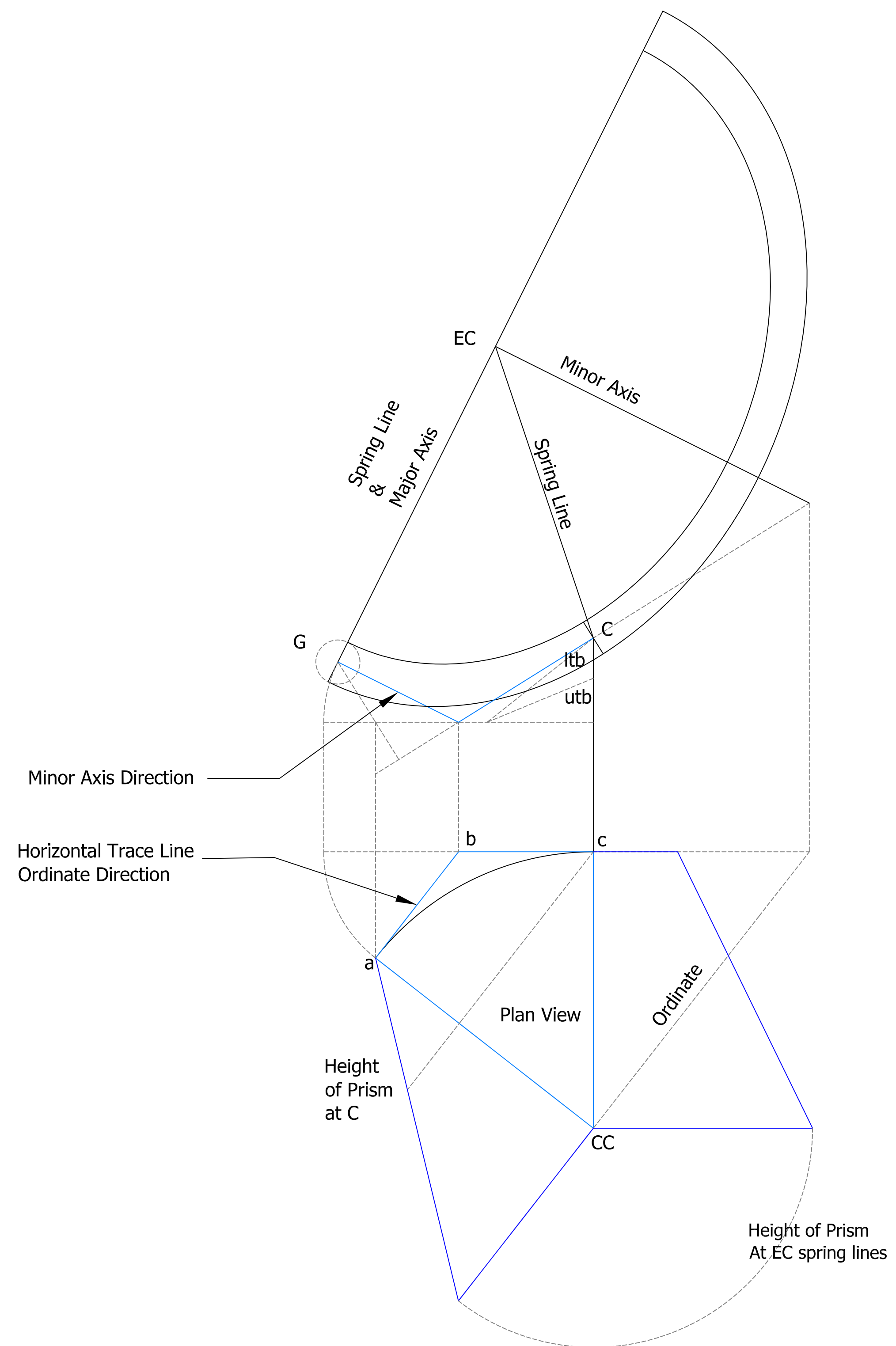


**Plate 9** Obtuse Plan, The Upper Tangent is Pitched The lower Tangent is Level



Plan = Obtuse  
 LowerTangent Pitch = 0.00°  
 Upper Tangent = 35.00°  
 Plan View Angle = 108.00°  
 Tangents in Original Position  
 Tangents shifted into Second Position



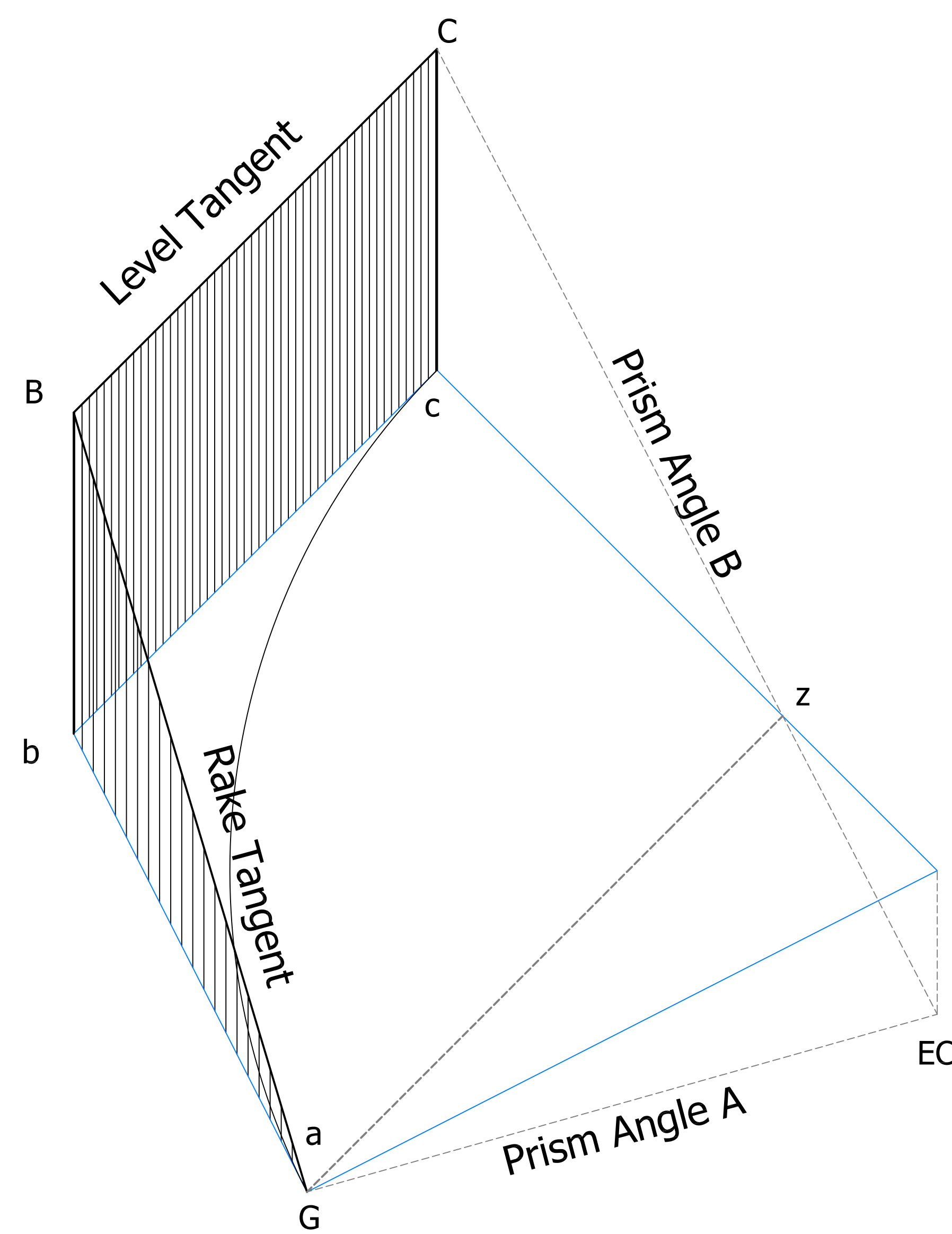


**Plate 9.1**

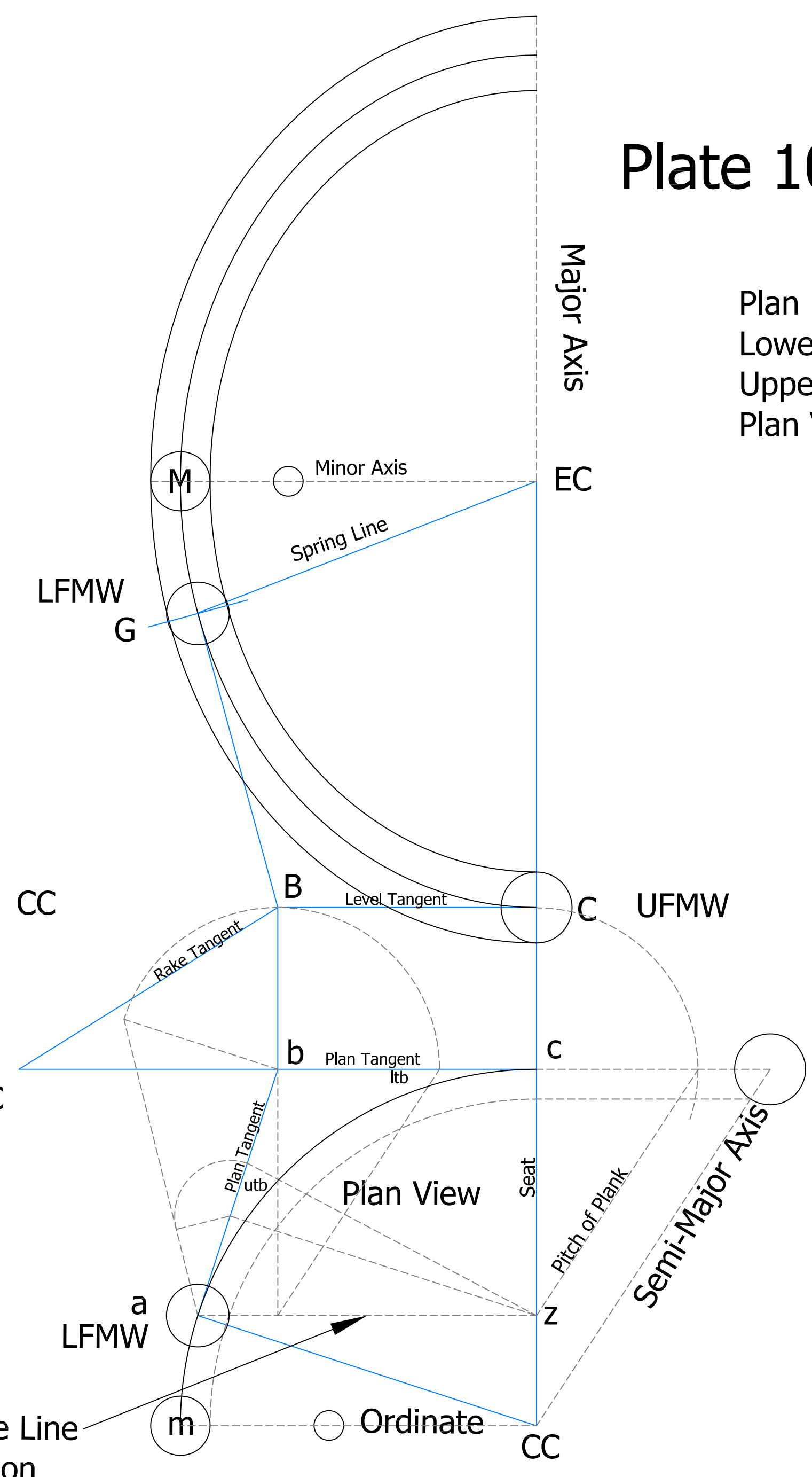
Obtuse Plan, The Upper Tangent is Pitched  
The lower Tangent is Level

Based on the book  
The Stair Builders Guide by Morris Williams 1914  
Manner Of Laying Out the Face Mould  
Fig 181.

Plan = Obtuse  
LowerTangent Pitch = 0.00°  
Upper Tangent = 32.00°  
Plan View Angle = 128.00

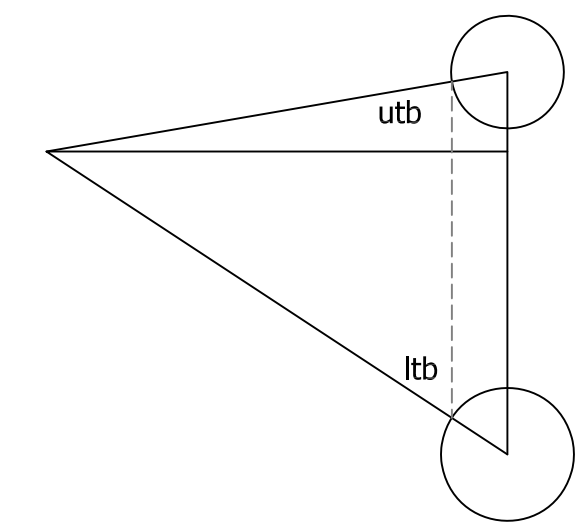


Isometric View of Prism

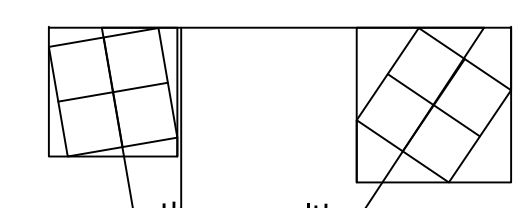


**Plate 10** Obtuse Plan, The Lower Tangent is Pitched  
The Upper Tangent is Level

Plan = Obtuse  
LowerTangent Pitch = 32.00°  
Upper Tangent = 0.00°  
Plan View Angle = 108.00

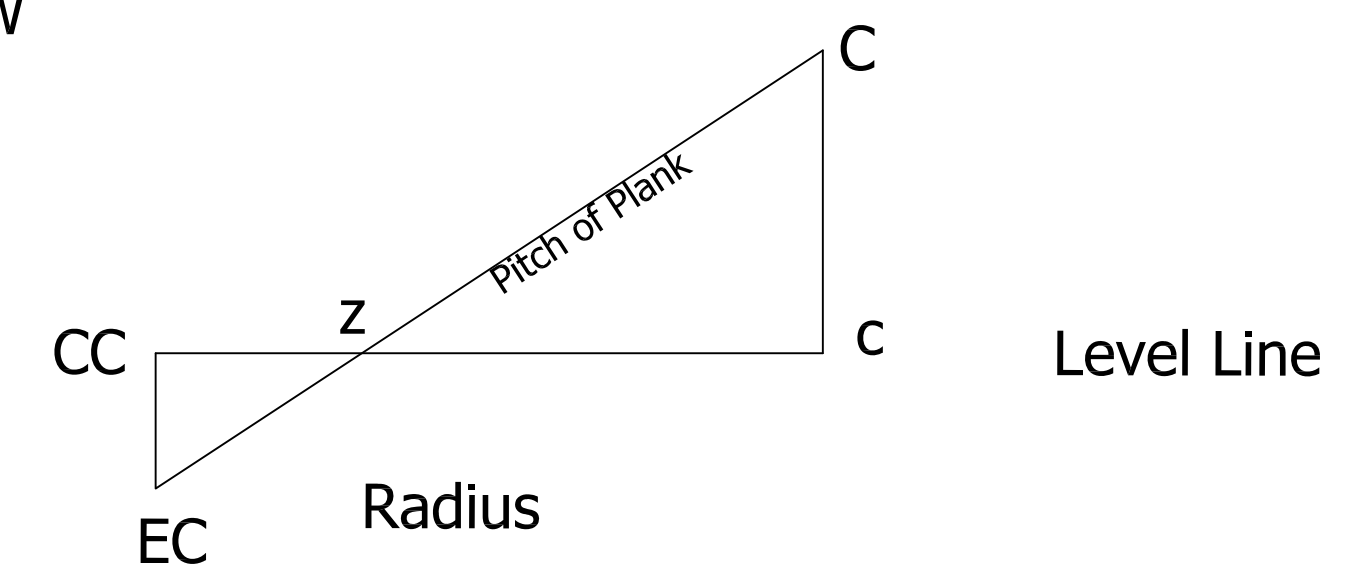


Rail Blocks for Square Cut Method

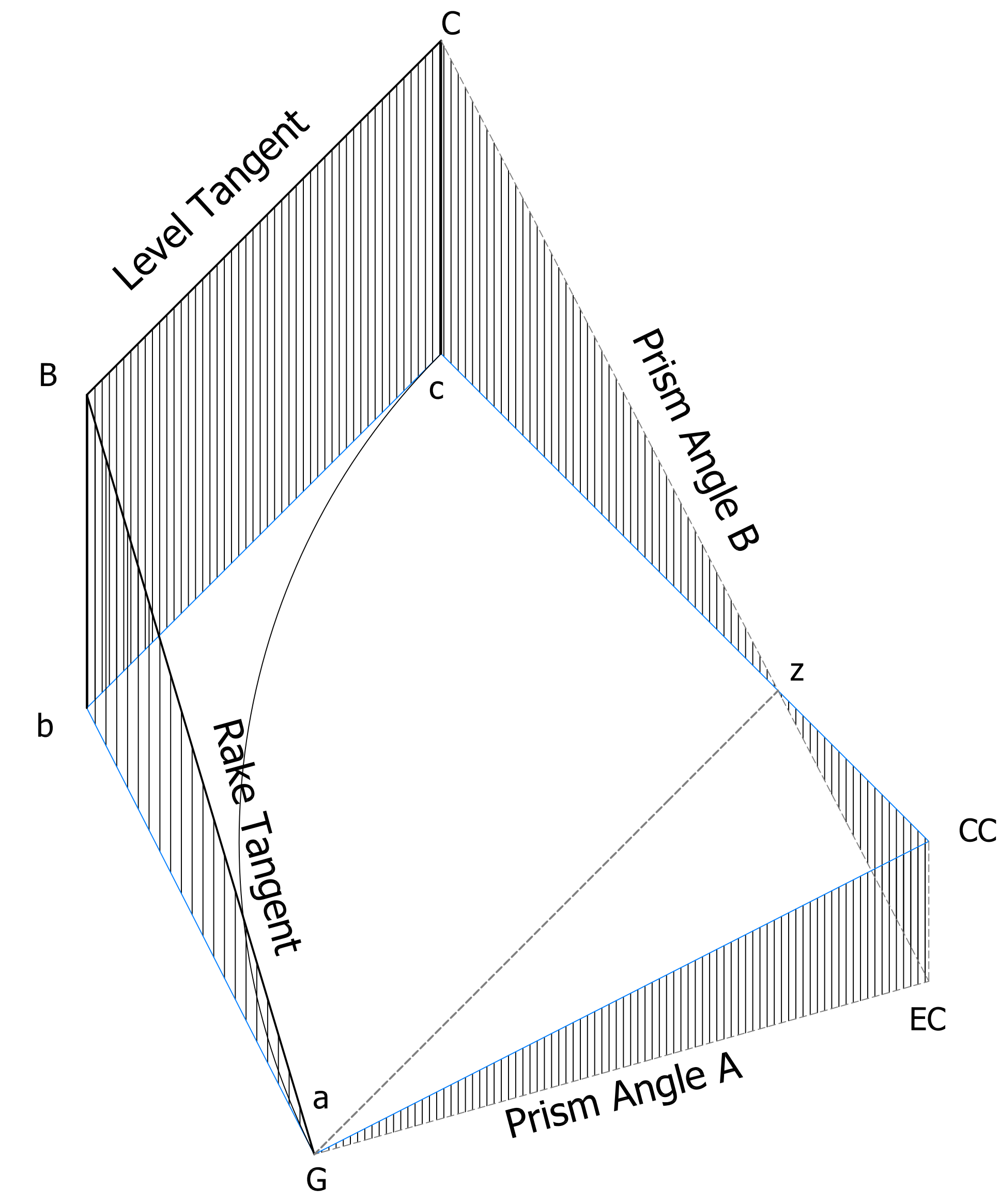


Height of Prism at C

UFMW



Height of Prism At EC spring lines is below the plan view



Isometric View of Prism

# Plate 11

## Acute Plan with Unequally Pitched Tangents

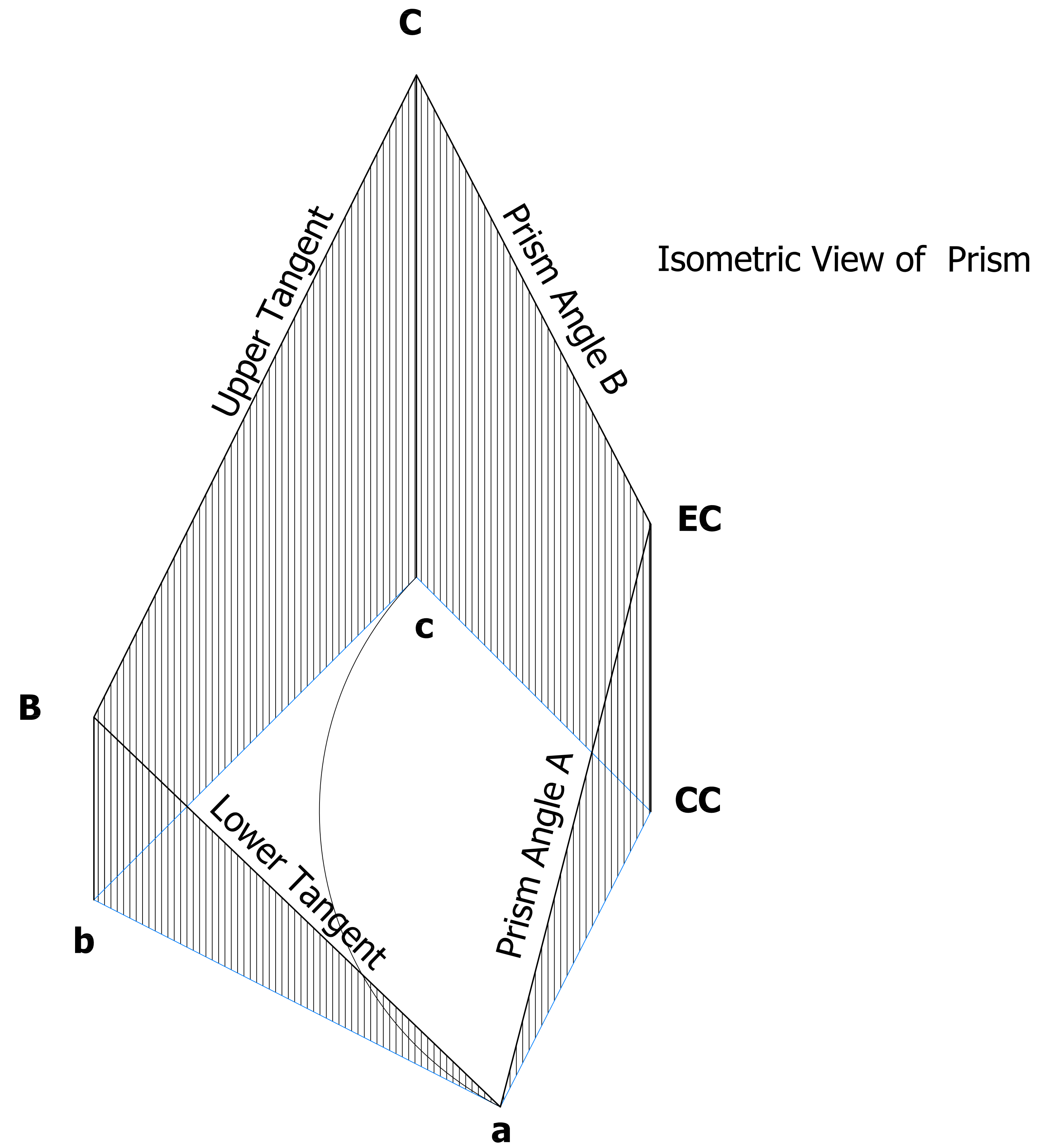
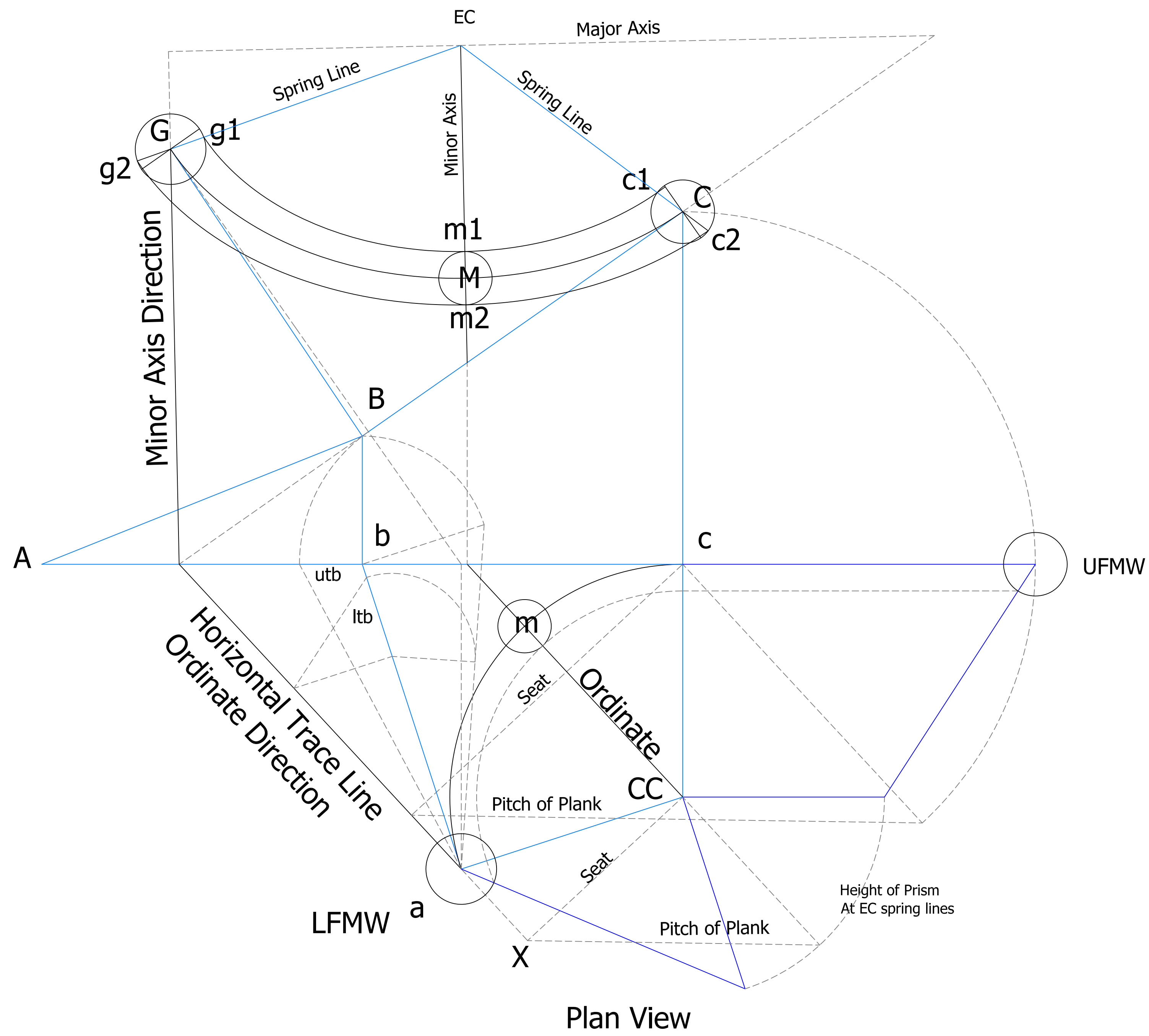




PLATE 16 Plan Acute with Short Upper Tangent Pitched

Face Mold width at Spring Lines

Tangent Shift for Face Mold at Upper Tangent

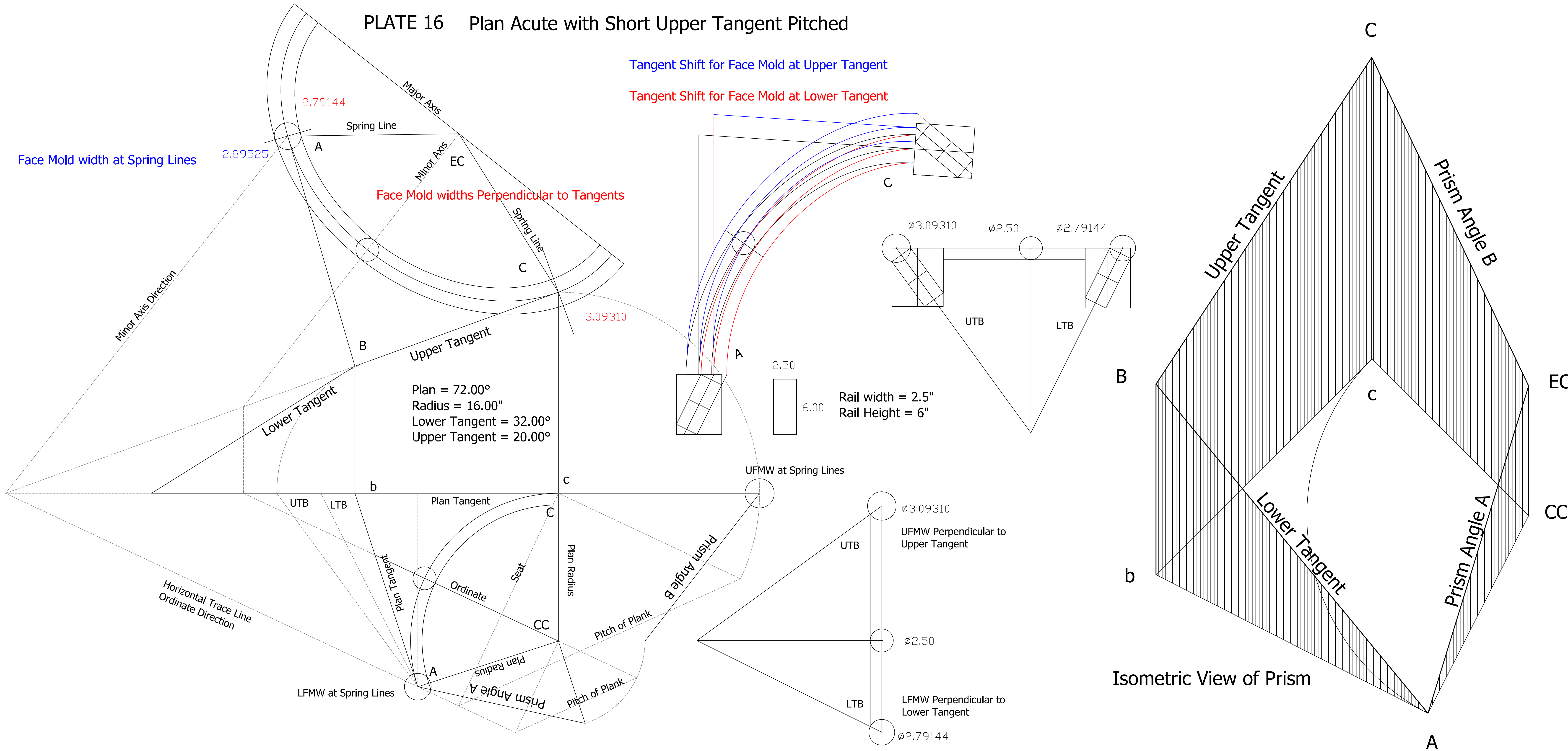
Tangent Shift for Face Mold at Lower Tangent

Face Mold widths Perpendicular to Tangents

Plan = 72.00°  
 Radius = 16.00"  
 Lower Tangent = 32.00°  
 Upper Tangent = 20.00°

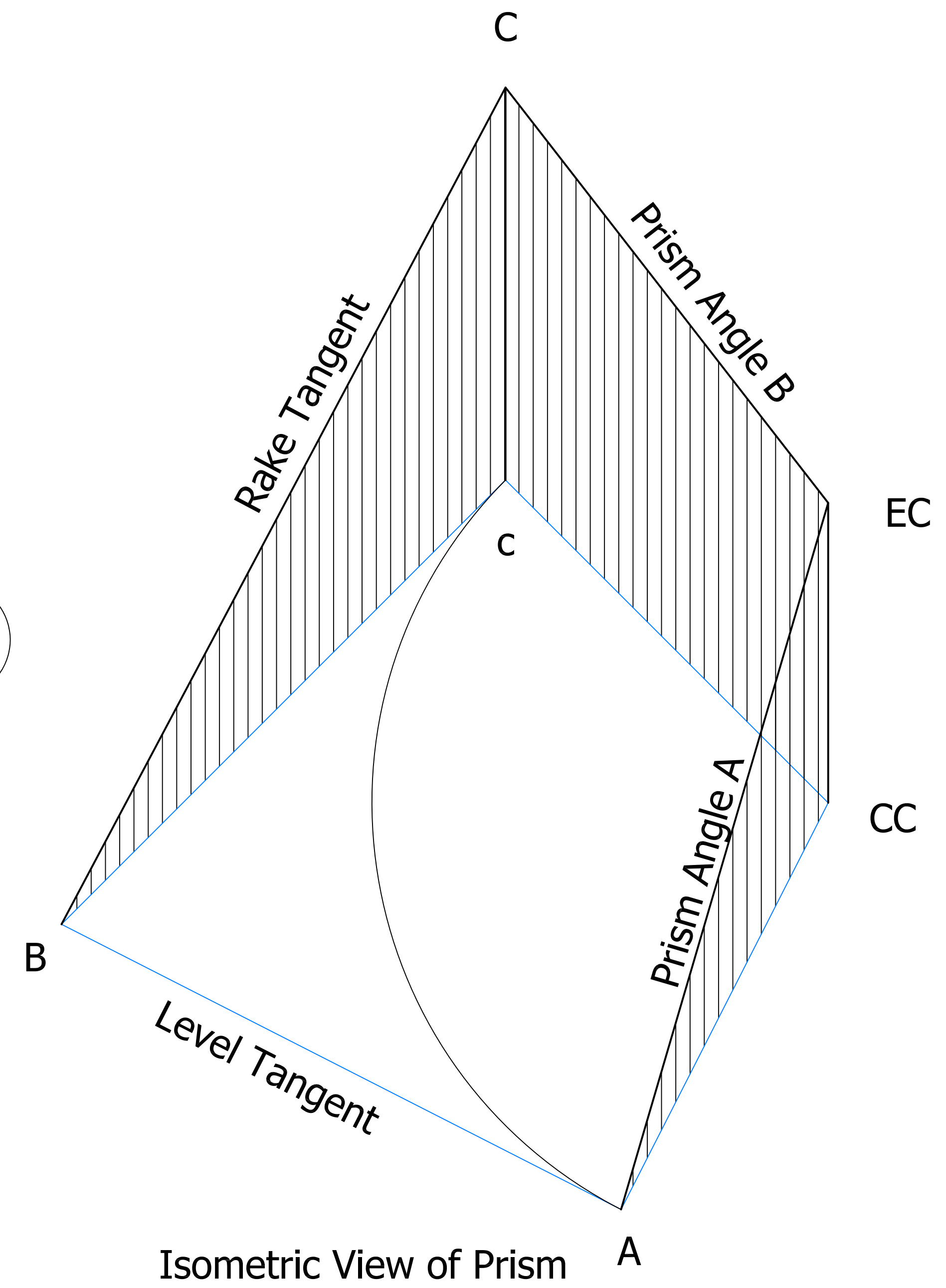
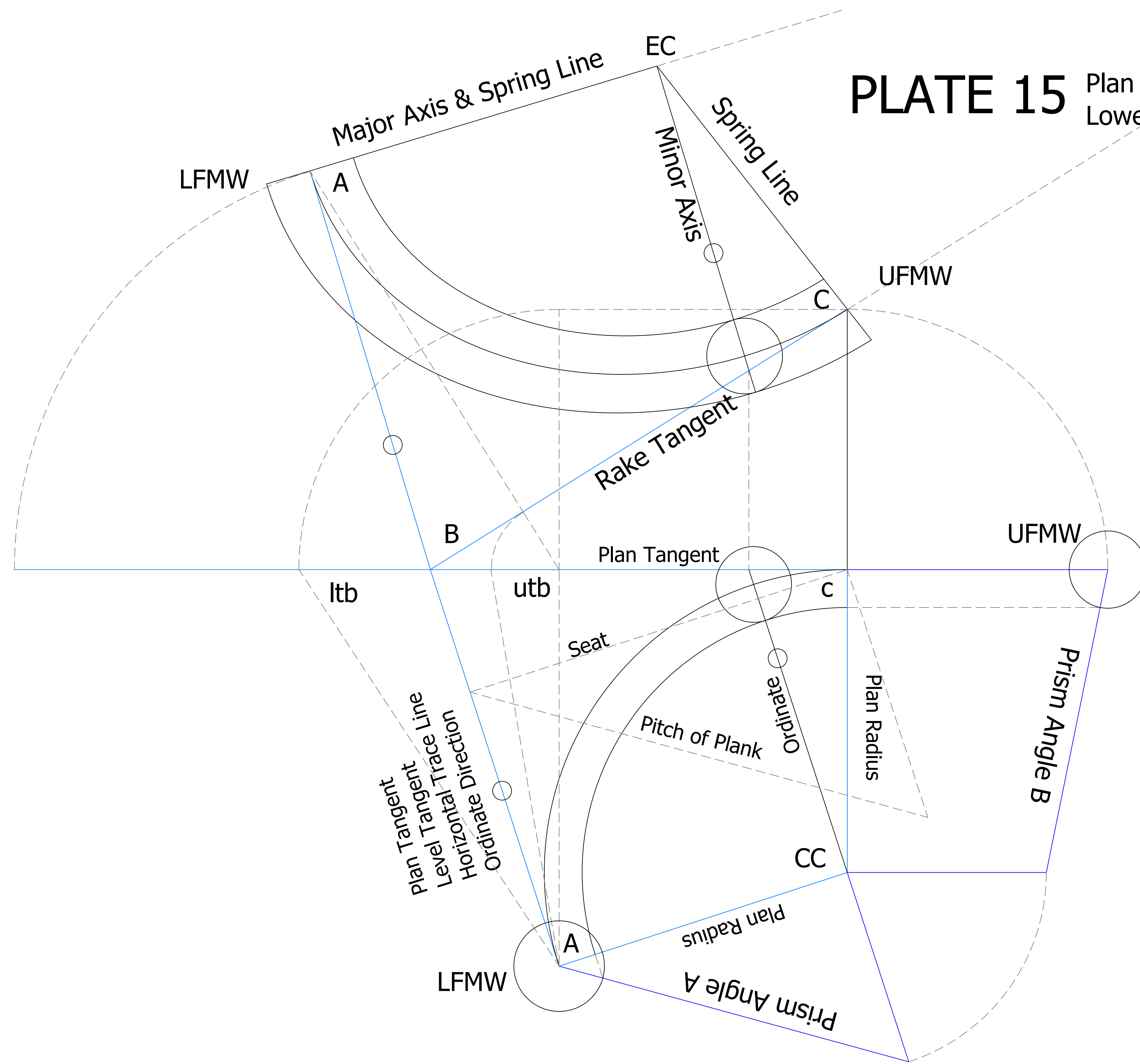
Rail width = 2.5"  
 Rail Height = 6"

Isometric View of Prism



# PLATE 15

Plan Acute with Upper Tangent Pitched  
Lower Tangent Level



# Plate 16

Quarter Circle Square Plan with Short Lower Pitched Tangent

## Plan View Section Plane Vertex Information

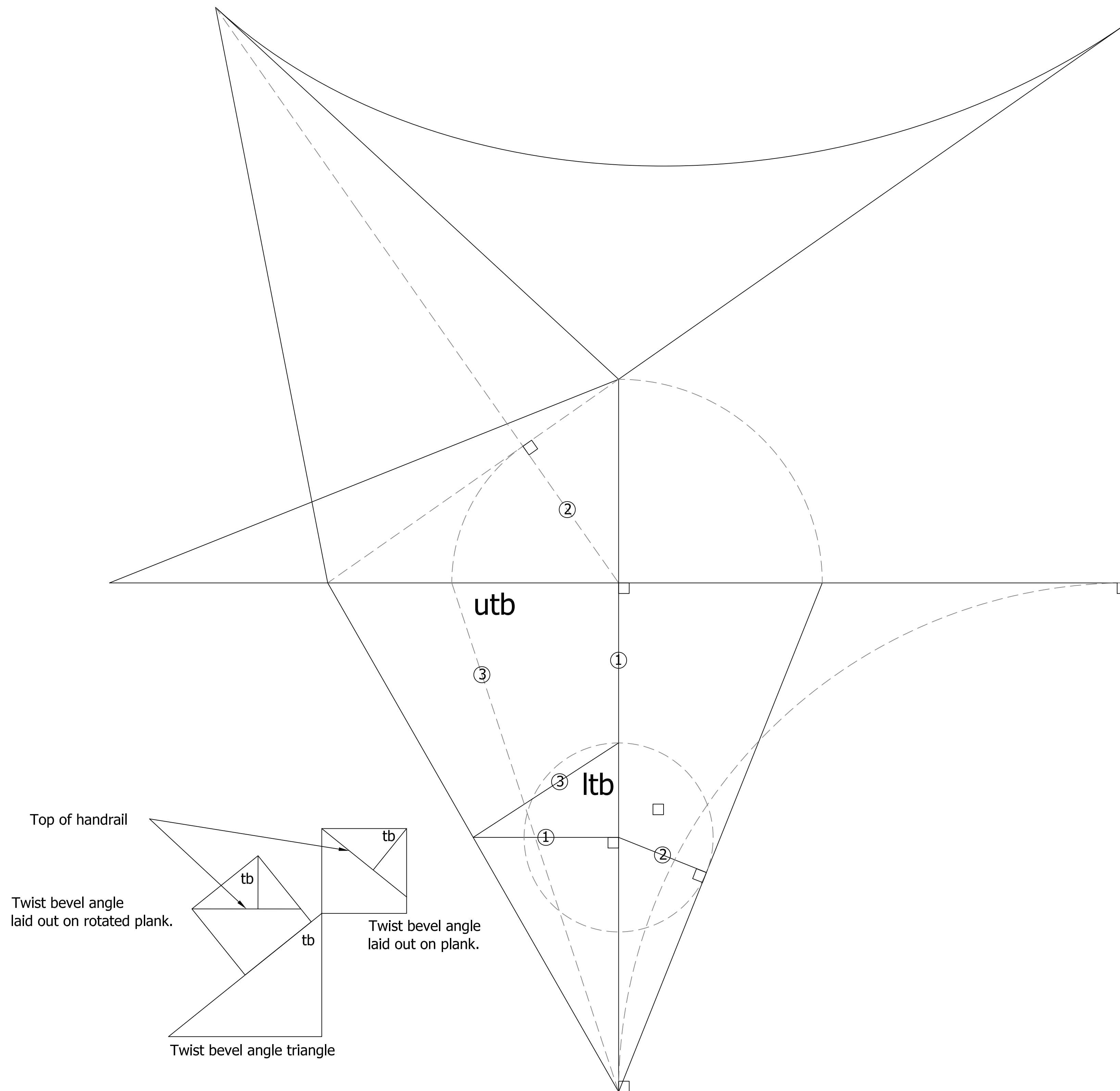
Plan = Square  
 Unequal Pitched Tangents  
 Plan View Angle = 90.00

## Tangent Angles

Upper Tangent Angle = 35.00000  
 Lower Tangent Angle = 21.80141  
 Upper Twist Bevel Angle = 71.85806  
 Lower Twist Bevel Angle = 56.97104

The Upper and Lower Twist Bevel Angles  
 are the Dihedral Angles between two Planes.

Line 1 is perpendicular to hip run  
 Line 2 is perpendicular to hip rafter  
 Line 3 triangle height = line 2 length



# Plate 17

Obtuse Plan with Short Lower Pitched Tangent

## Plan View Section Plane Vertex Information

Plan = Obtuse

Unequal Pitched tangents

Plan View Angle = 108.00000

## Tangent Angles

Upper Tangent Angle = 35.00000

Lower Tangent Angle = 21.80141

Upper Twist Bevel Angle = 81.01273

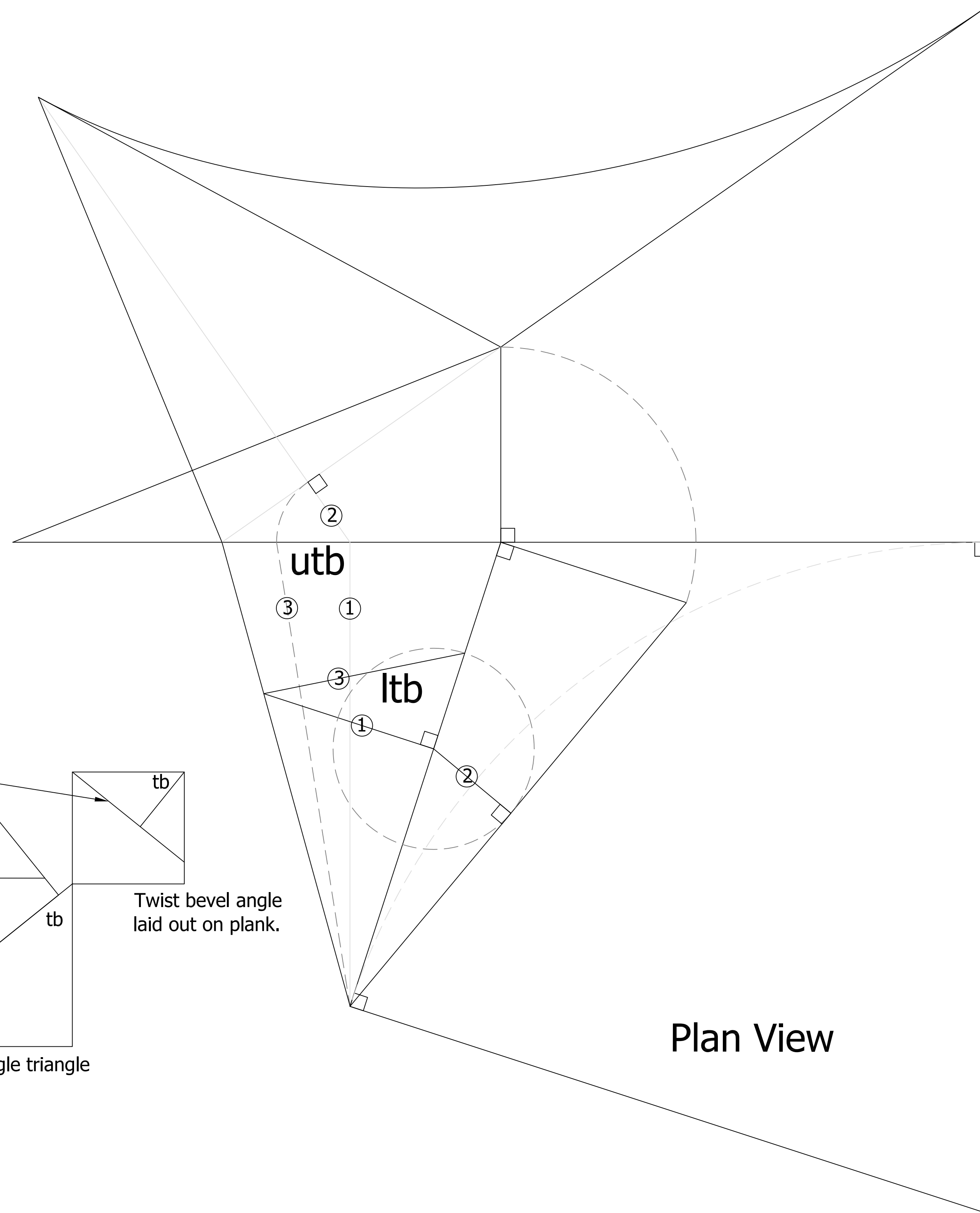
Lower Twist Bevel Angle = 60.62435

The Upper and Lower Twist Bevel Angles  
are the Dihedral Angles between two Planes.

Line 1 is perpendicular to hip run

Line 2 is perpendicular to hip rafter

Line 3 triangle height = line 2 length



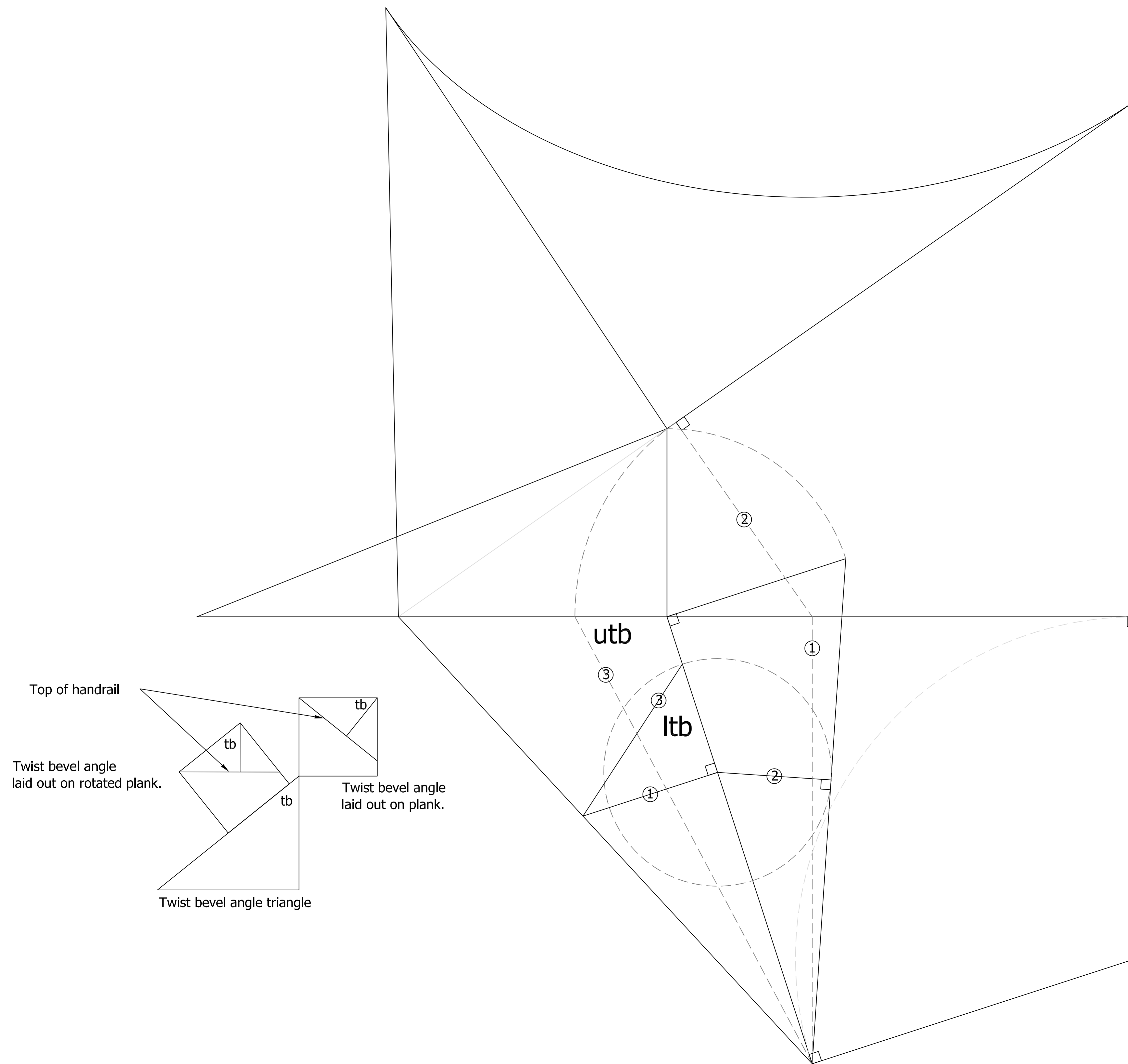
Top of handrail

Twist bevel angle  
laid out on rotated plank.

Twist bevel angle  
laid out on plank.

Twist bevel angle triangle

Plan View



# Plate 18

Acute Plan with Short Lower Pitched Tangent

## Plan View Section Plane Vertex Information

Plan = Acute

Unequal Pitched Tangents

Plan View Angle = 72.00

## Tangent Angles

Upper Tangent Angle = 35.00000

Lower Tangent Angle = 21.80141

Upper Twist Bevel Angle = 62.03665

Lower Twist Bevel Angle = 51.19183

The Upper and Lower Twist Bevel Angles are the Dihedral Angles between two Planes.

Line 1 is perpendicular to hip run

Line 2 is perpendicular to hip rafter

Line 3 triangle height = line 2 length

Plate 19.1 Tangent Handrail folding template for oblique plane angles.

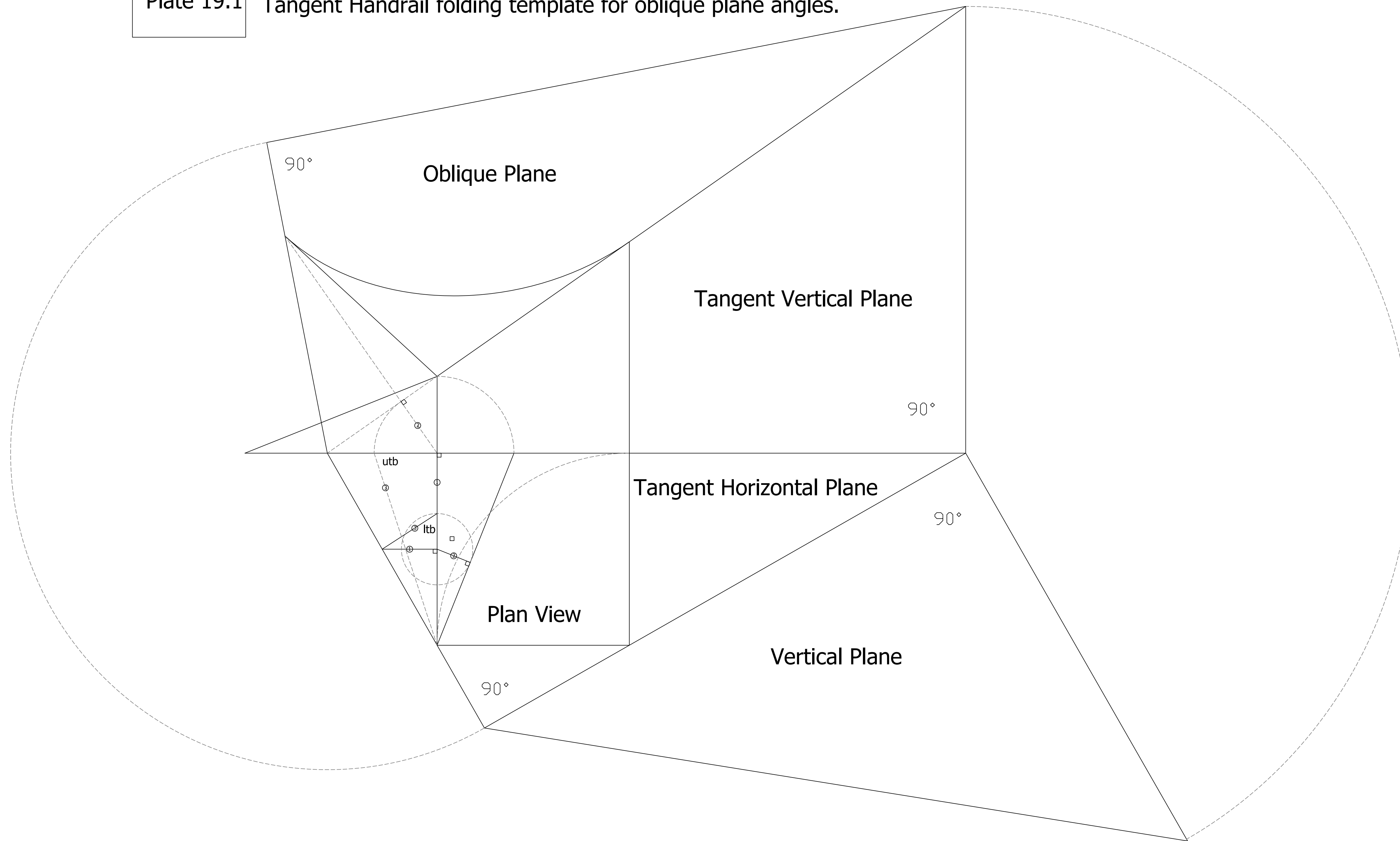


Plate 19.2 Minor Axis Offset

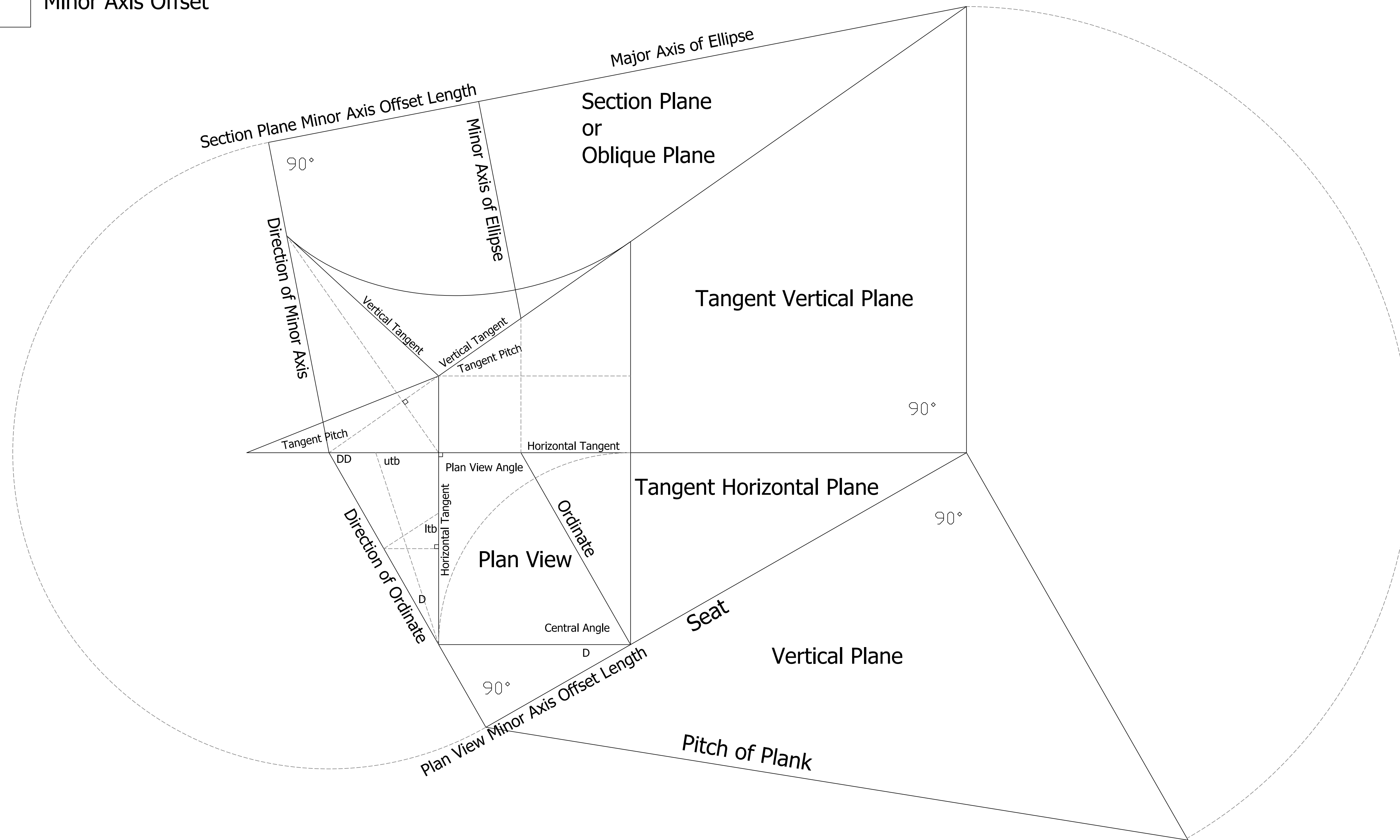
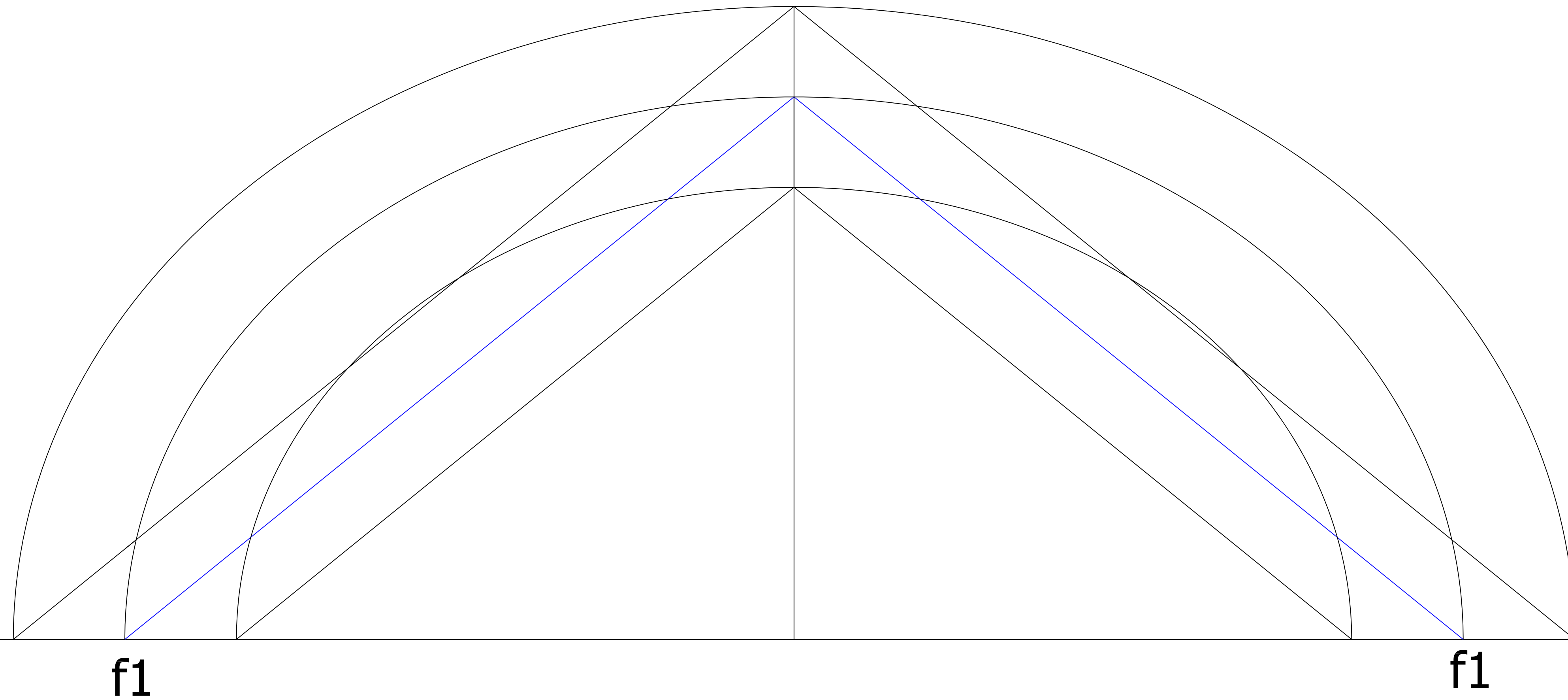


Plate 19.3



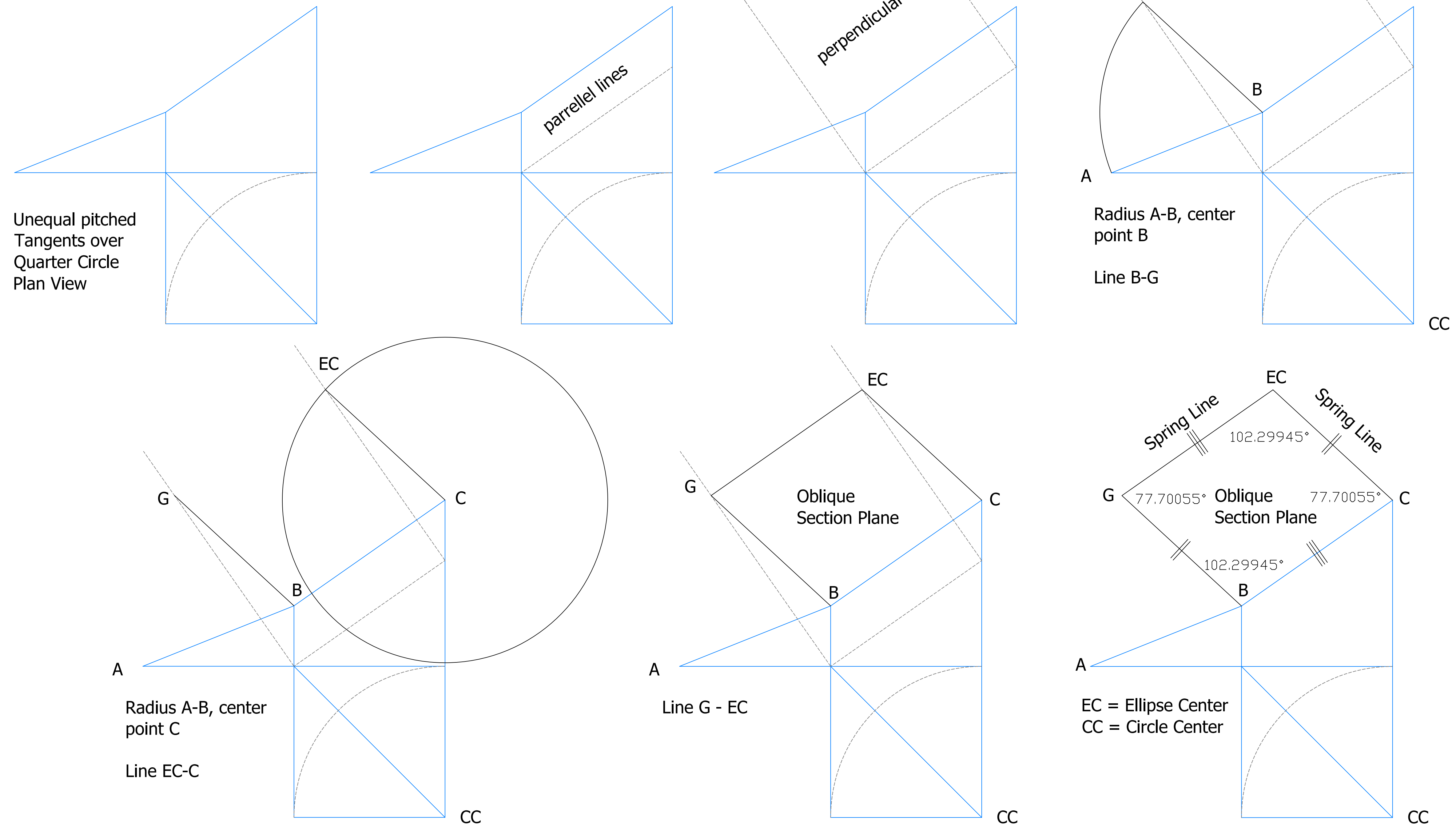
Face Mould Ellipse semi major axis lines are parallel

Draw face mould semi major axis lines for center of face mould (blue), then draw semi major axis lines that are parallel to the semi major axis lines of the center of the face mould for the inner and outer curves of the face mould.

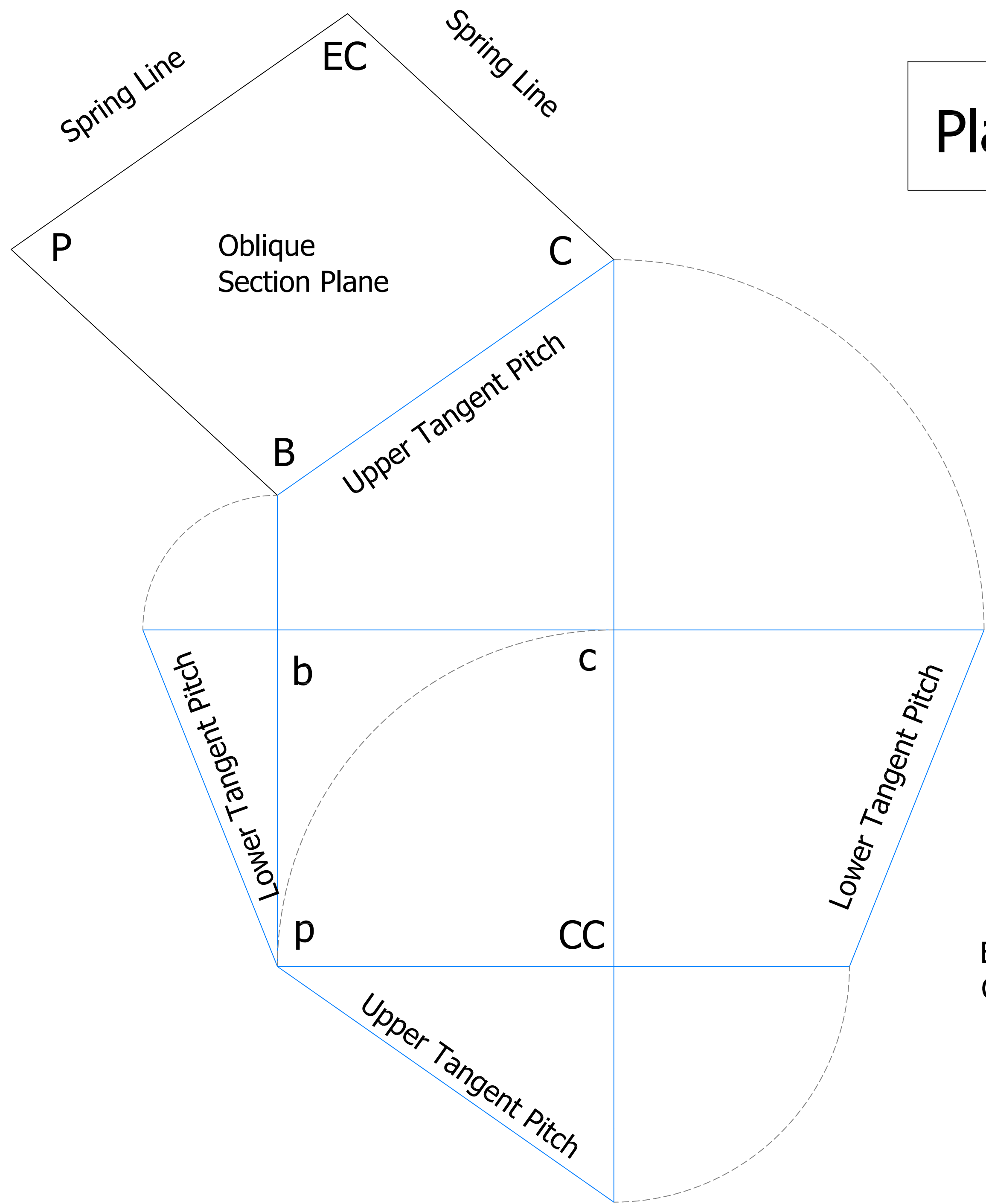


Plate 19.4

Tangent Handrail drawing for oblique section plane and center of ellipse for quarter circle plans.

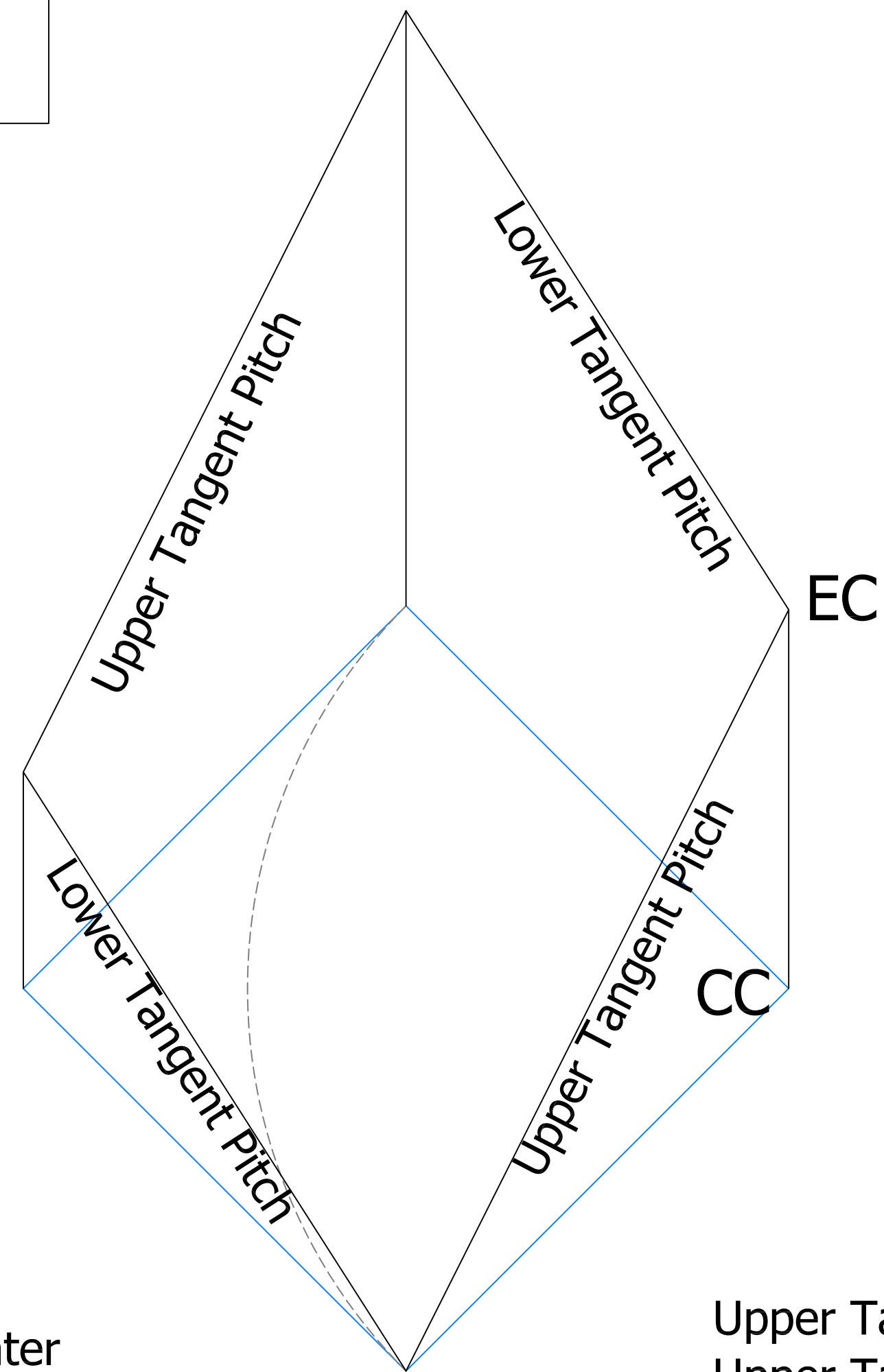


Net of prism for tangent handrailing for Quarter Circle with Unequal Pitched Tangents



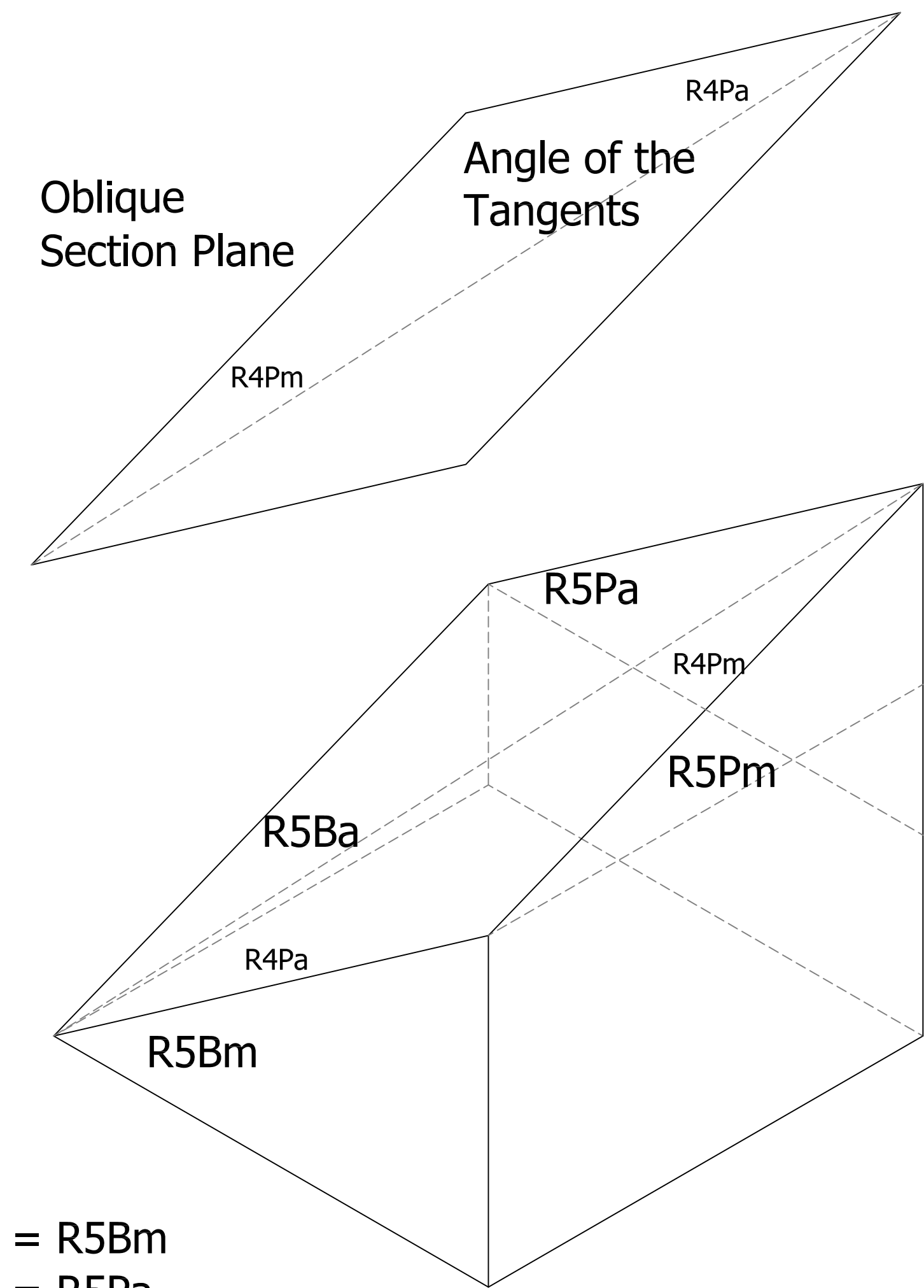
Isometric drawing of prism for tangent handrailing for Quarter Circle with Unequal Pitched Tangents

**Plate 19.6**

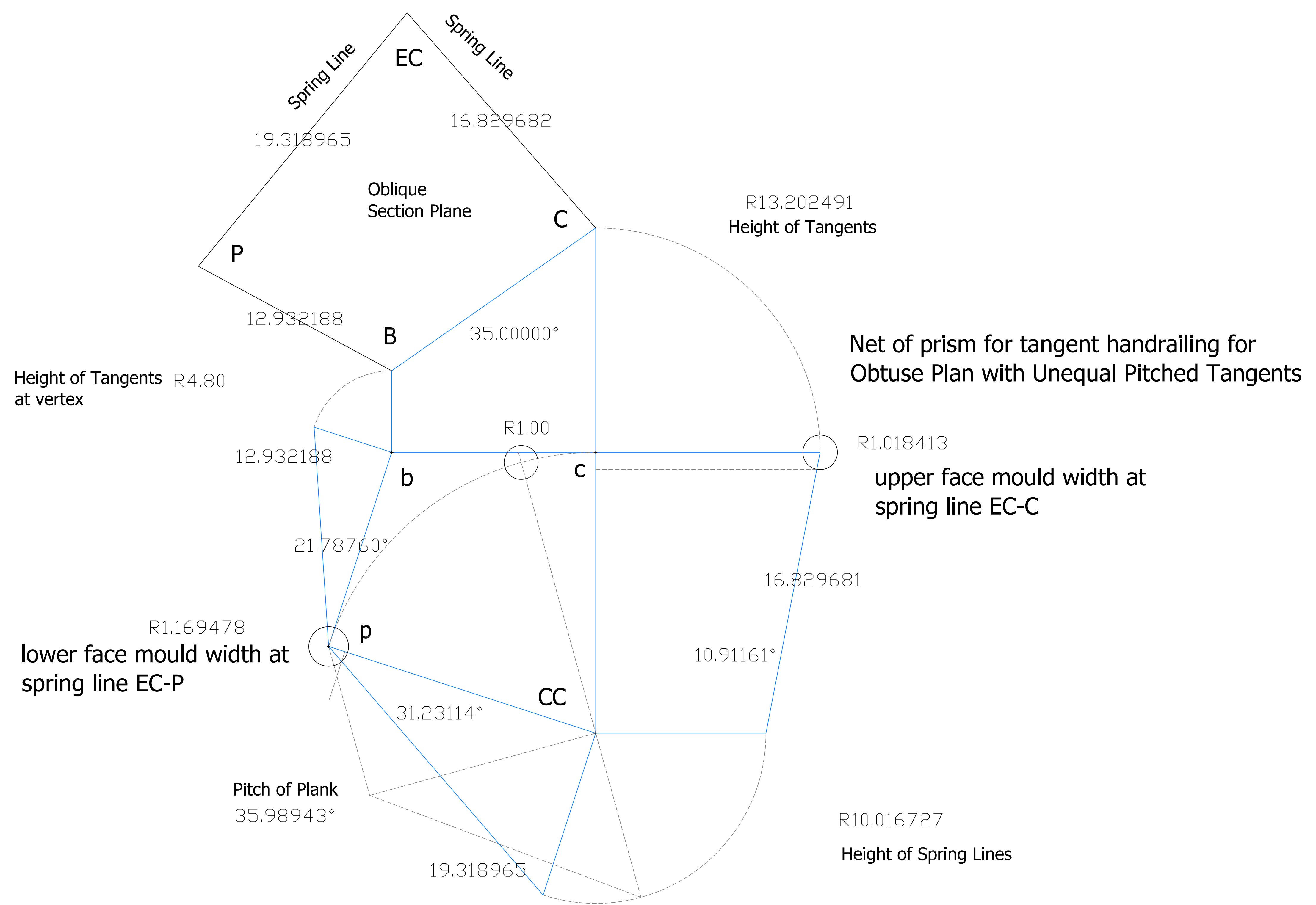


EC = Ellipse Center  
CC = Circle Center

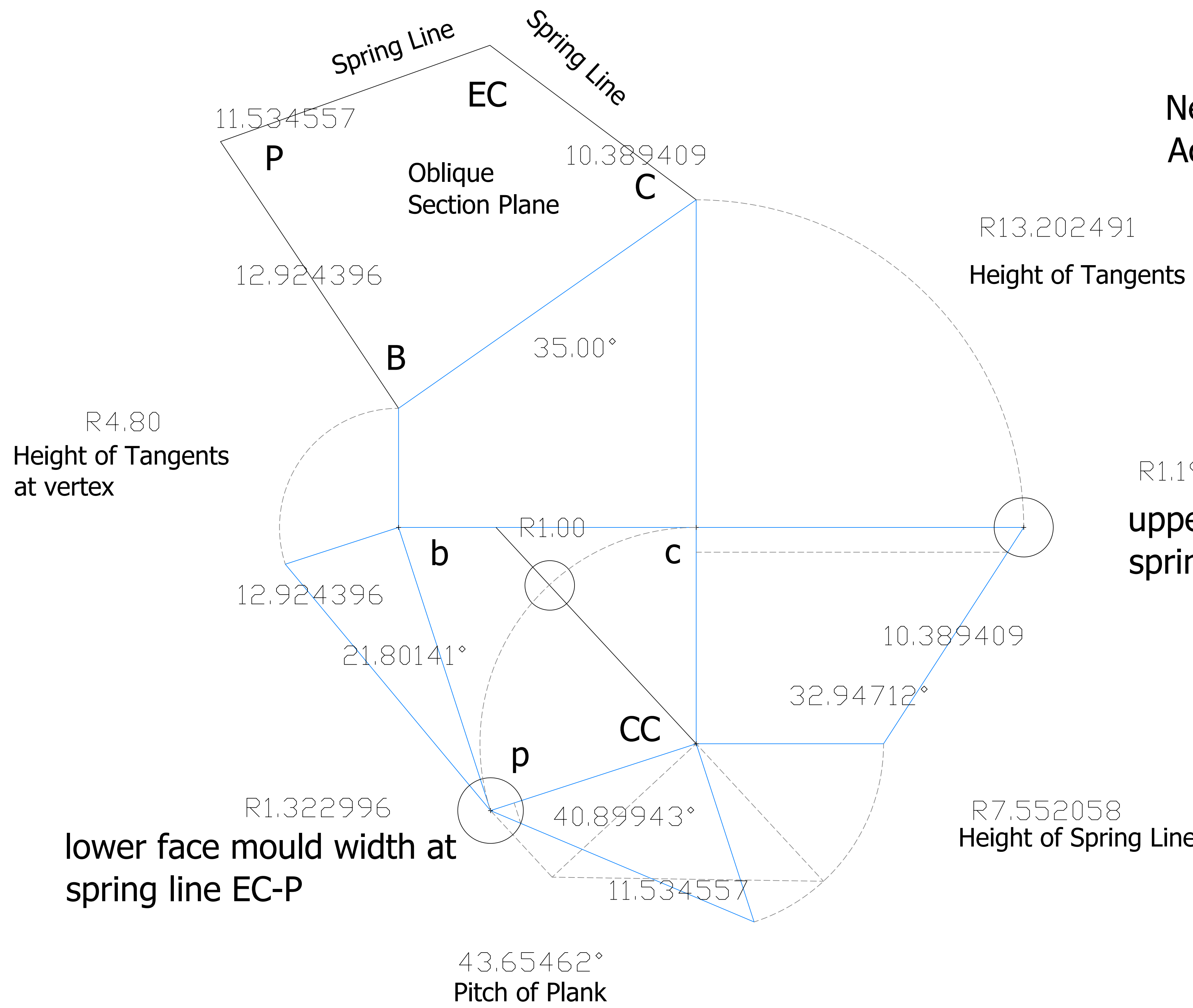
Isometric drawing of prism for tangent handrailing for Quarter Circle with Unequal Pitched Tangents showing the relationship to roof framing geometry and trigonometry for Hip Rafter Diamond Post



Upper Tangent Pitch = R5Bm  
Upper Tangent Pitch = R5Pa  
Lower Tangent Pitch = R5Ba  
Lower Tangent Pitch = R5Pm  
Angle of the Tangents =  $180^\circ - R4Pm - R4Pa$



Net of prism for tangent handrailing for Acute Plan with Unequal Pitched Tangents



R1.191650  
upper face mould width at spring line EC-C

lower face mould width at spring line EC-P

Height of Spring Lines

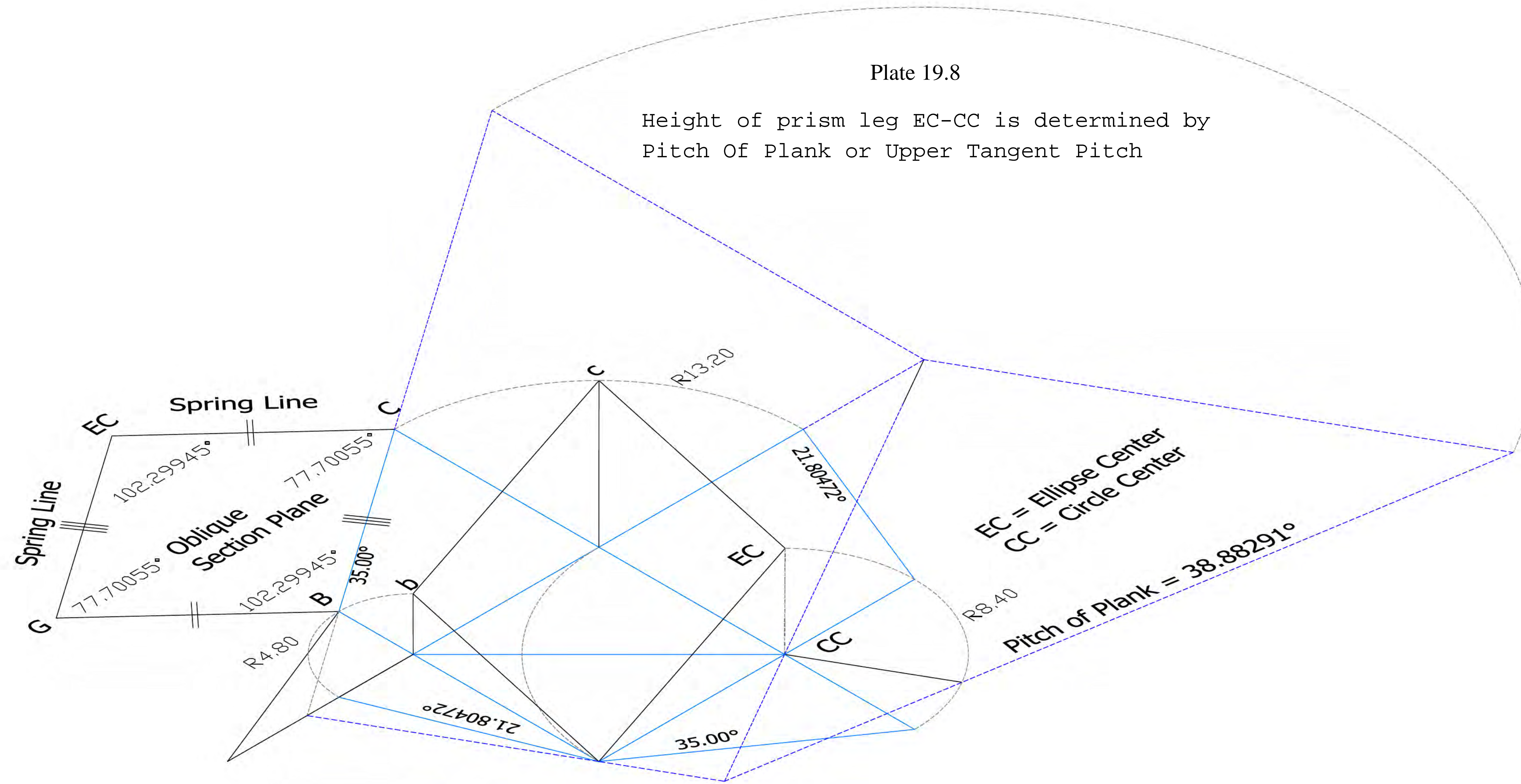
Height of Tangents at vertex

Height of Tangents

Pitch of Plank

Plate 19.8

Height of prism leg EC-CC is determined by  
Pitch Of Plank or Upper Tangent Pitch



# Plate 20

Acute Plan with Short Lower Pitched Tangent

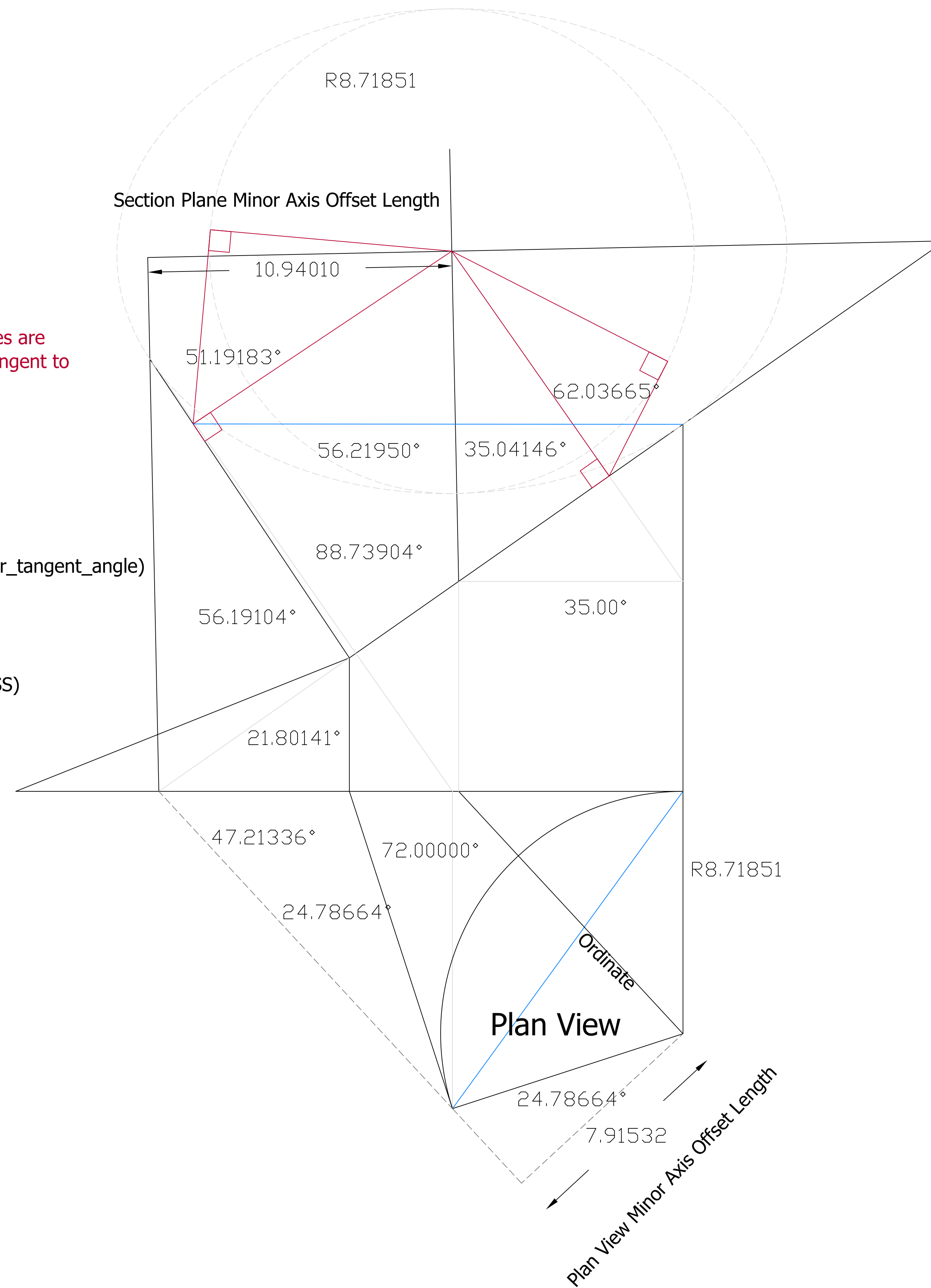
Upper & Lower Twist Bevel Angles are perpendicular to tangents and tangent to right cylinder on section plane

$$\text{slant\_angle} = \text{upper\_tangent\_angle}$$

$$\begin{aligned} (\text{Angle\_D}, \text{Angle\_DD}) &= \text{get\_plan\_angles}(\text{plan\_view\_angle}, \text{upper\_tangent\_angle}, \text{lower\_tangent\_angle}) \\ \text{SS} &= \arctan(\tan(\text{slant\_angle}) \div \sin(\text{Angle\_DD})) \\ \text{S} &= \arctan(\tan(\text{slant\_angle}) \div \sin(\text{Angle\_D})) \end{aligned}$$

$$\begin{aligned} \text{Plan View Minor Axis Offset Length} &= \text{radius} * \cos(\text{Angle\_D}) \\ \text{Section Plane Minor Axis Offset Length} &= \text{Plan View Minor Axis Offset Length} \div \cos(\text{SS}) \\ \text{major\_axis} &= \text{radius} \div \cos(\text{SS}) \\ \text{utb} &= 90^\circ - \arctan(\sin(\text{upper\_tangent\_angle}) \div \tan(\text{Angle\_DD})) \\ \text{lbt} &= 90^\circ - \arctan(\sin(\text{lower\_tangent\_angle}) \div \tan(\text{Angle\_D})) \end{aligned}$$

$$\begin{aligned} \text{R1} &= \arctan(\tan(\text{upper\_tangent\_angle}) \div \sin(\text{Angle\_DD})) \\ \text{R4Bm} &= \arctan(\cos(\text{R1}) \div \tan(\text{Angle\_DD})) \\ \text{R4Ba} &= \arctan(\cos(\text{R1}) \div \tan(\text{Angle\_D})) \\ \text{Angle of the Tangents} &= 180 - \text{R4Bm} - \text{R4Ba} \end{aligned}$$



Plan View Section Plane Vertex Information  
 Plan View Angle = 72.0000000000  
 Plan View Radius = 8.7185100000

Tangent Angles  
 Upper Tangent Angle = 35.0000000000  
 Lower Tangent Angle = 21.8014100000  
 Upper Twist Bevel Angle = 62.0366474261  
 Lower Twist Bevel Angle = 51.1918289162

Ellipse Angles  
 Plan Angle DD = 47.2133581426  
 Plan Angle D = 24.7866418574  
 Minor Axis = 8.7185100000  
 Major Axis = 12.0502272720  
 Plan View Minor Axis Offset Length = 7.9153194146  
 Section Plane Minor Axis Offset Length = 10.9401030539  
 R1 = 43.65462  
 R4Bm = 33.80896  
 R4Ba = 57.45200  
 Angle of the Tangents = 180 - 33.80896 - 57.45200 = 88.73904

Tetrahedron Angles

D Angle = 47.2133581426  
 A Angle = 43.6546170252  
 C Angle = 35.0000000000  
 E Angle = 33.8089621505  
 B Angle = 27.9633525739

90-D Angle = 42.7866418574  
 90-A Angle = 46.3453829748  
 90-C Angle = 55.0000000000  
 90-E Angle = 56.1910378495  
 90-B Angle = 62.0366474261

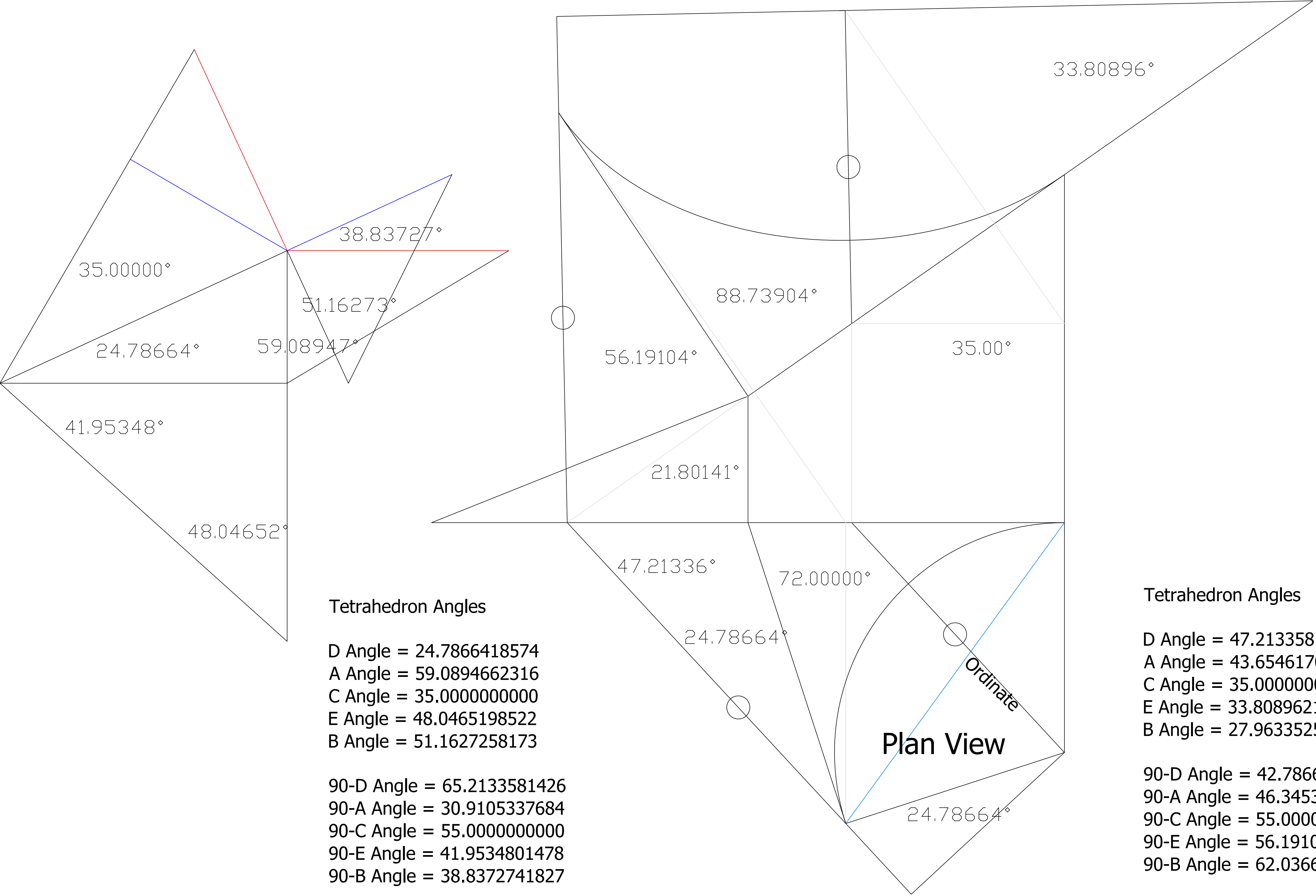
Tetrahedron Angles

D Angle = 24.7866418574  
 A Angle = 59.0894662316  
 C Angle = 35.0000000000  
 E Angle = 48.0465198522  
 B Angle = 51.1627258173

90-D Angle = 65.2133581426  
 90-A Angle = 30.9105337684  
 90-C Angle = 55.0000000000  
 90-E Angle = 41.9534801478  
 90-B Angle = 38.8372741827

# Plate 21

Acute Plan with Short Lower Pitched Tangent



**Tetrahedron Angles**

- D Angle = 24.7866418574
- A Angle = 59.0894662316
- C Angle = 35.0000000000
- E Angle = 48.0465198522
- B Angle = 51.1627258173
  
- 90-D Angle = 65.2133581426
- 90-A Angle = 30.9105337684
- 90-C Angle = 55.0000000000
- 90-E Angle = 41.9534801478
- 90-B Angle = 38.8372741827

**Tetrahedron Angles**

- D Angle = 47.2133581426
- A Angle = 43.6546170252
- C Angle = 35.0000000000
- E Angle = 33.8089621505
- B Angle = 27.9633525739
  
- 90-D Angle = 42.7866418574
- 90-A Angle = 46.3453829748
- 90-C Angle = 55.0000000000
- 90-E Angle = 56.1910378495
- 90-B Angle = 62.0366474261

Plan View Section Plane Vertex Information  
 Plan = Acute  
 Unequal Pitched Tangents  
 Plan View Angle = 72.00

Tangent Angles  
 Upper Tangent Angle = 35.00  
 Lower Tangent Angle = 21.80141  
 Upper Twist Bevel Angle = 62.0366474261  
 Lower Twist Bevel Angle = 51.1918289162

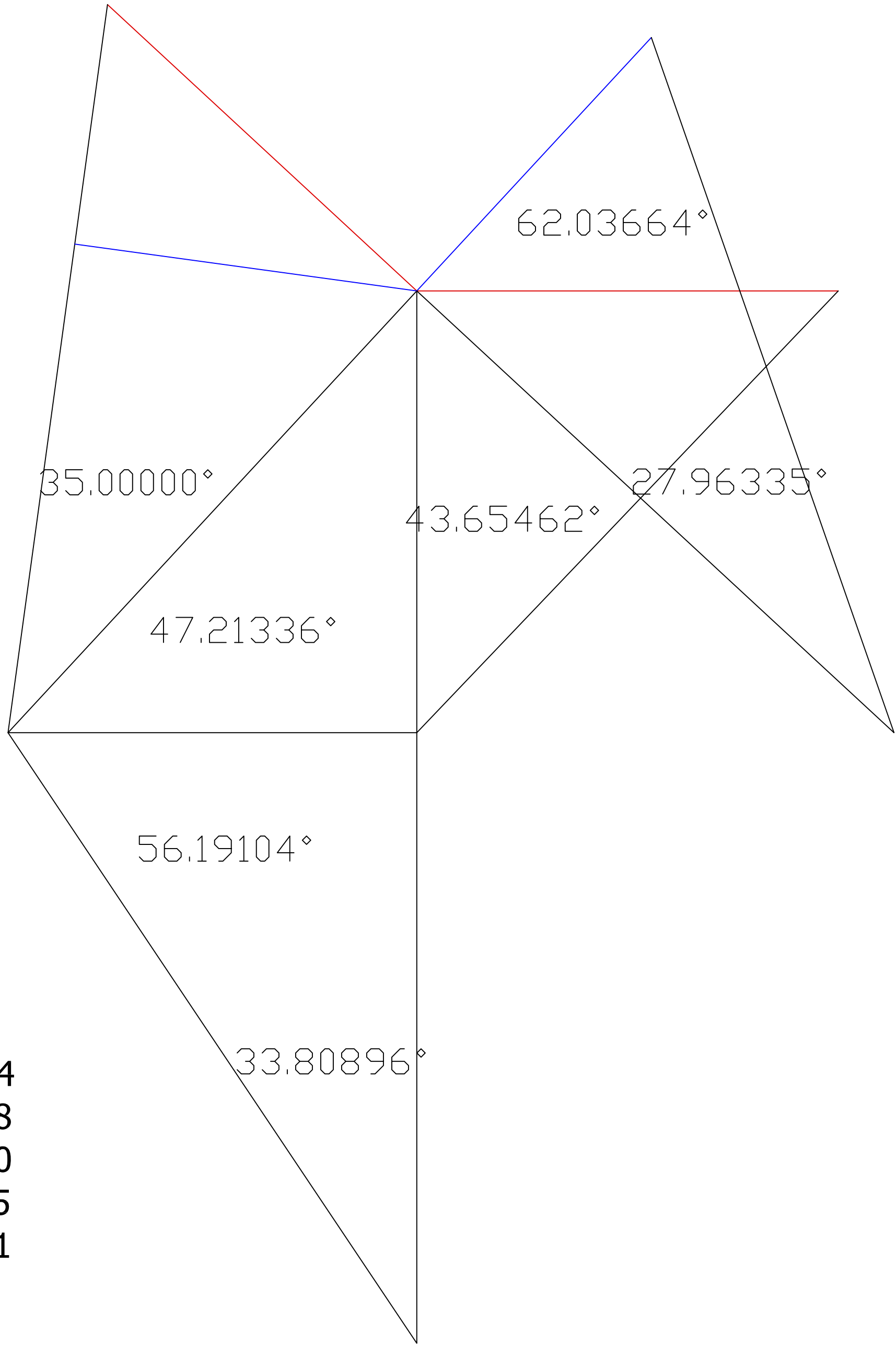
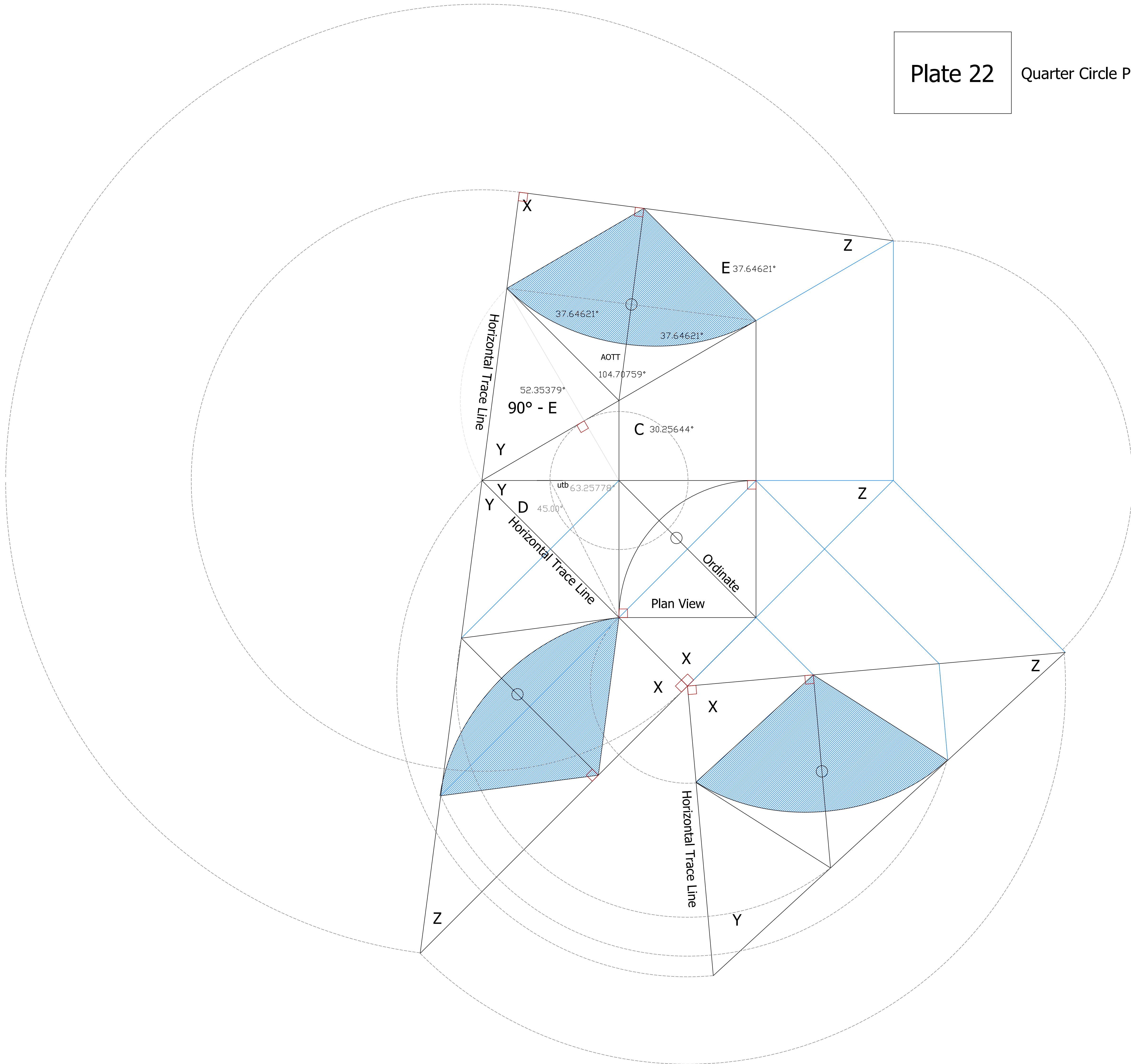


Plate 22

Quarter Circle Plan with Equally Pitched Tangents



Plan View Section Plane Vertex Information

Plan = Square  
 Equal Pitched Tangents  
 Plan View Angle = 90.00000  
 Plan View Radius = 12.00000

Tangent Angles

Upper Tangent Angle = 30.25644  
 Lower Tangent Angle = 30.25644  
 Upper Twist Bevel Angle = 63.25779  
 Lower Twist Bevel Angle = 63.25779

Dihedral Angle at Upper Tangent Angle = 63.25779

R1 = 39.52120  
 R4Bm = 37.64621  
 R4Ba = 37.64621  
 Angle of the Tangents =  $180^\circ - 37.64621^\circ - 37.64621^\circ = 104.70759^\circ$

Ellipse Angles

Plan Angle DD = 45.00000  
 Plan Angle D = 45.00000  
 Minor Axis = 12.00000  
 Major Axis = 15.55635  
 Plan View Minor Axis Offset Length = 8.48528  
 Section Plane Minor Axis Offset Length = 11.00000

Tetrahedron Angles

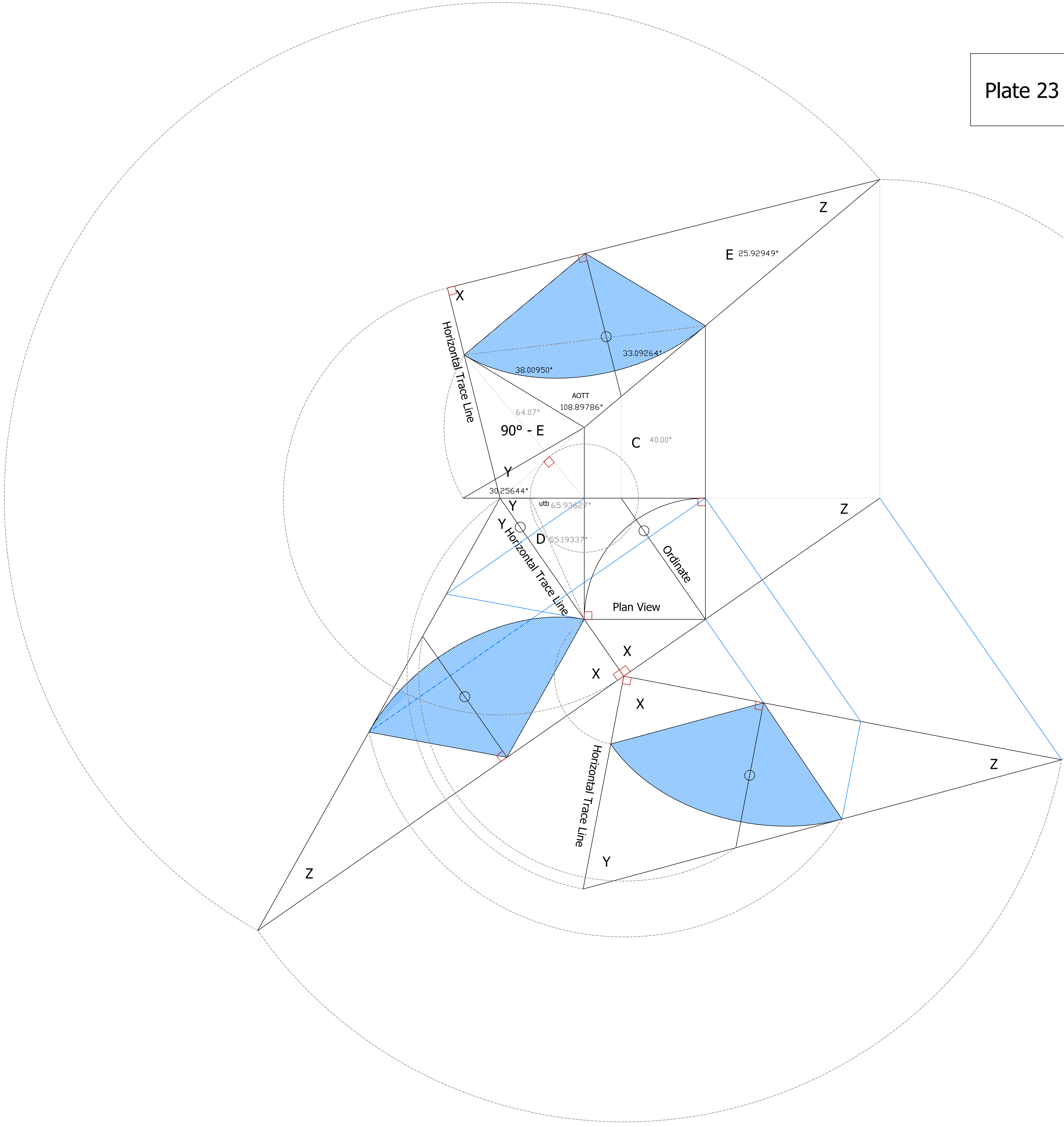
D Angle = 45.00000  
 A Angle = 39.52120  
 C Angle = 30.25644  
 E Angle = 37.64621  
 B Angle = 26.74221

90-D Angle = 45.00000  
 90-A Angle = 50.47880  
 90-C Angle = 59.74356  
 90-E Angle = 52.35379  
 90-B Angle = 63.25779



**Plate 23**

Quarter Circle Plan with Shorter Pitched Tangent



Plan View Section Plane Vertex Information

Plan = Square  
 Unequal Pitched Tangents  
 Plan View Angle = 90.00000  
 Plan View Radius = 12.00000

Tangent Angles  
 Upper Tangent Angle = 40.00000  
 Lower Tangent Angle = 30.25644  
 Upper Twist Bevel Angle = 65.92208  
 Lower Twist Bevel Angle = 54.06553

Dihedral Angle at Upper Tangent Angle = 65.92208

R1 = 45.62175  
 R4Bm = 25.92949  
 R4Ba = 45.17265  
 Angle of the Tangents =  $180^\circ - 25.92949^\circ - 45.17265^\circ = 108.89786^\circ$

Ellipse Angles  
 Plan Angle DD = 55.19337  
 Plan Angle D = 34.80663  
 Minor Axis = 12.00000  
 Major Axis = 17.15776  
 Plan View Minor Axis Offset Length = 9.85300  
 Section Plane Minor Axis Offset Length = 14.08795

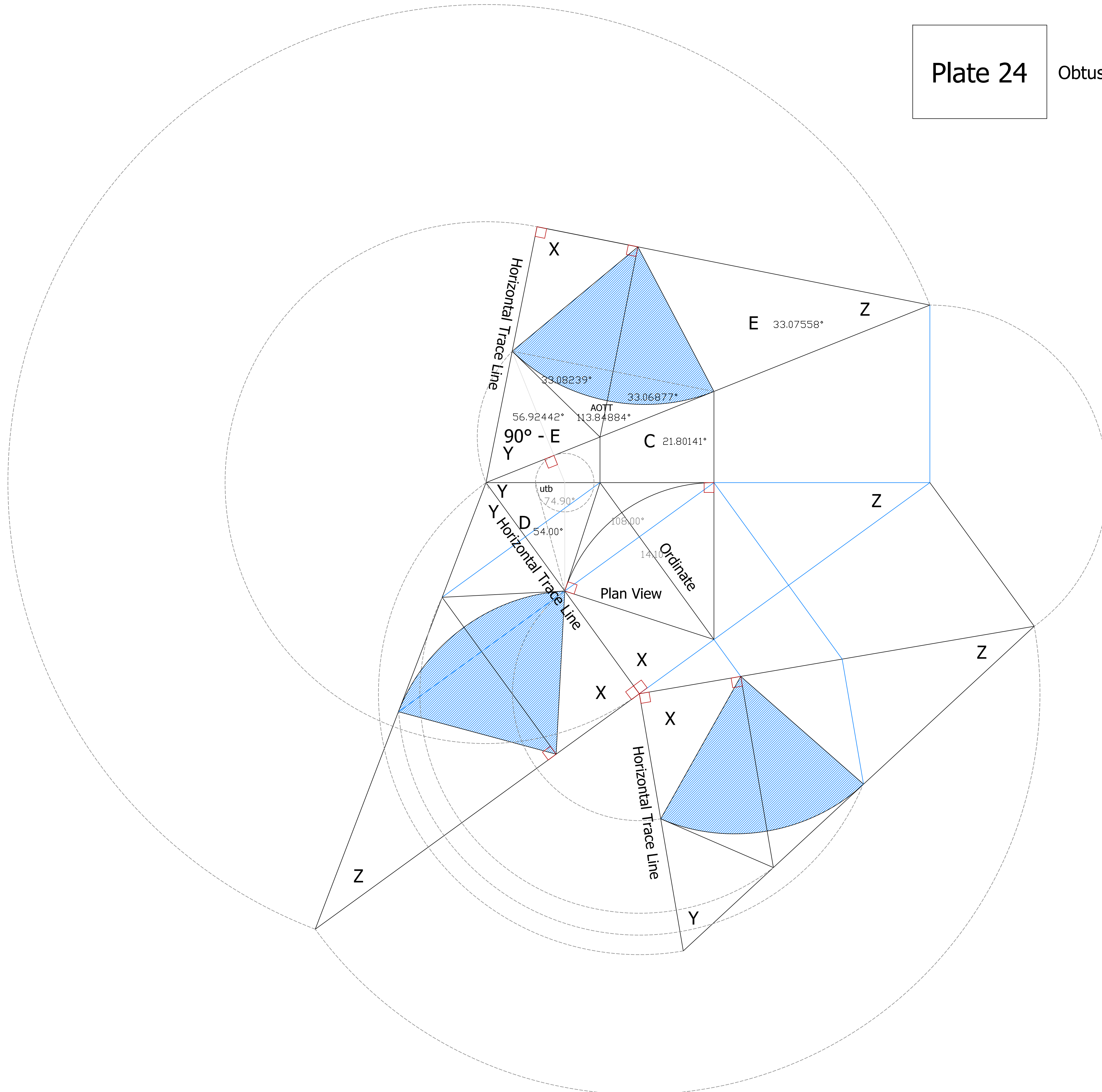
Tetrahedron Angles

D Angle = 55.19337	D Angle = 34.80663
A Angle = 45.62175	A Angle = 55.77399
C Angle = 40.00000	C Angle = 40.00000
E Angle = 25.92949	E Angle = 38.97537
B Angle = 24.07792	B Angle = 42.75715

90-D Angle = 34.80663	90-D Angle = 55.19337
90-A Angle = 44.37825	90-A Angle = 34.22601
90-C Angle = 50.00000	90-C Angle = 50.00000
90-E Angle = 64.07051	90-E Angle = 51.02463
90-B Angle = 65.92208	90-B Angle = 47.24285

**Plate 24**

Obtuse Plan with Equally Pitched Tangents



Plan View Section Plane Vertex Information

Plan = Obtuse  
 Equal Pitched Tangents  
 Plan View Angle = 108.00000  
 Plan View Radius = 8.71851

Tangent Angles

Upper Tangent Angle = 21.80141  
 Lower Tangent Angle = 21.80141  
 Upper Twist Bevel Angle = 74.89944  
 Lower Twist Bevel Angle = 74.89944

Dihedral Angle at Upper Tangent Angle = 74.89944

R1 = 26.30904  
 R4Bm = 33.07558  
 R4Ba = 33.07558  
 Angle of the Tangents =  $180^\circ - 33.07558^\circ - 33.07558^\circ = 113.84884^\circ$

Ellipse Angles

Plan Angle DD = 54.00000  
 Plan Angle D = 54.00000  
 Minor Axis = 8.71851  
 Major Axis = 9.72596  
 Plan View Minor Axis Offset Length = 5.12461  
 Section Plane Minor Axis Offset Length = 5.71678

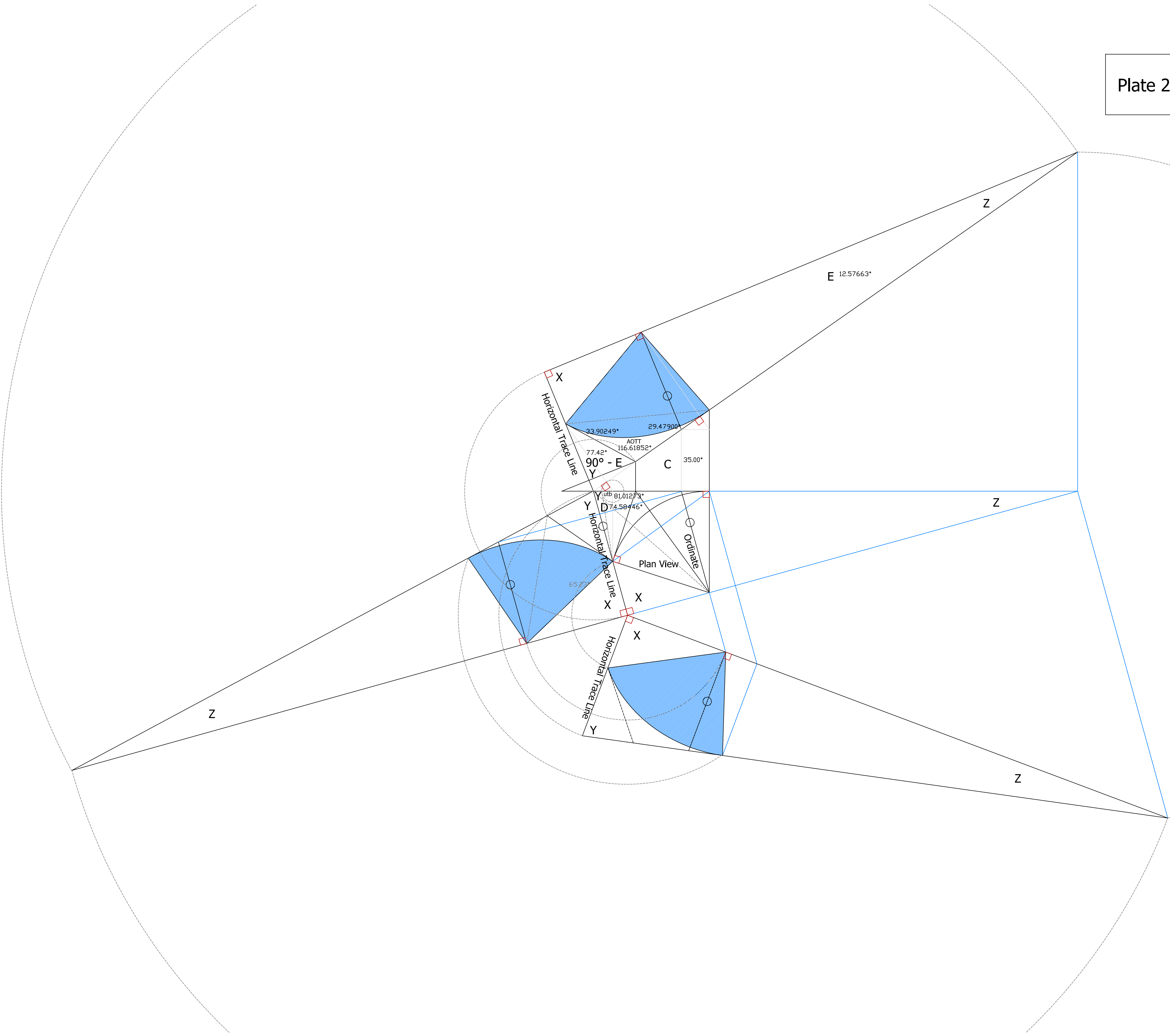
Tetrahedron Angles

D Angle = 54.00000  
 A Angle = 26.30904  
 C Angle = 21.80141  
 E Angle = 33.07558  
 B Angle = 15.10056

90-D Angle = 36.00000  
 90-A Angle = 63.69096  
 90-C Angle = 68.19859  
 90-E Angle = 56.92442  
 90-B Angle = 74.89944

**Plate 25**

Obtuse Plan with Shorter Pitched Tangent



Plan View Section Plane Vertex Information  
 Plan = Obtuse  
 Unequal Pitched tangents  
 Plan View Angle = 108.00000  
 Plan View Radius = 8.71851

Tangent Angles  
 Upper Tangent Angle = 35.00000  
 Lower Tangent Angle = 21.80141  
 Upper Twist Bevel Angle = 81.01273  
 Lower Twist Bevel Angle = 60.62435

Dihedral Angle at Upper Tangent Angle = 81.01273

R1 = 35.99236  
 R4Bm = 12.57663  
 R4Ba = 50.80486  
 Angle of the Tangents =  $180^\circ - 12.57663^\circ - 50.80486^\circ = 116.61852^\circ$

Ellipse Angles  
 Plan Angle DD = 74.58446  
 Plan Angle D = 33.41554  
 Minor Axis = 8.71851  
 Major Axis = 10.77563  
 Plan View Minor Axis Offset Length = 7.27733  
 Section Plane Minor Axis Offset Length = 8.99440

Tetrahedron Angles  
 D Angle = 74.58446  
 A Angle = 35.99236  
 C Angle = 35.00000  
 E Angle = 12.57663  
 B Angle = 8.98727

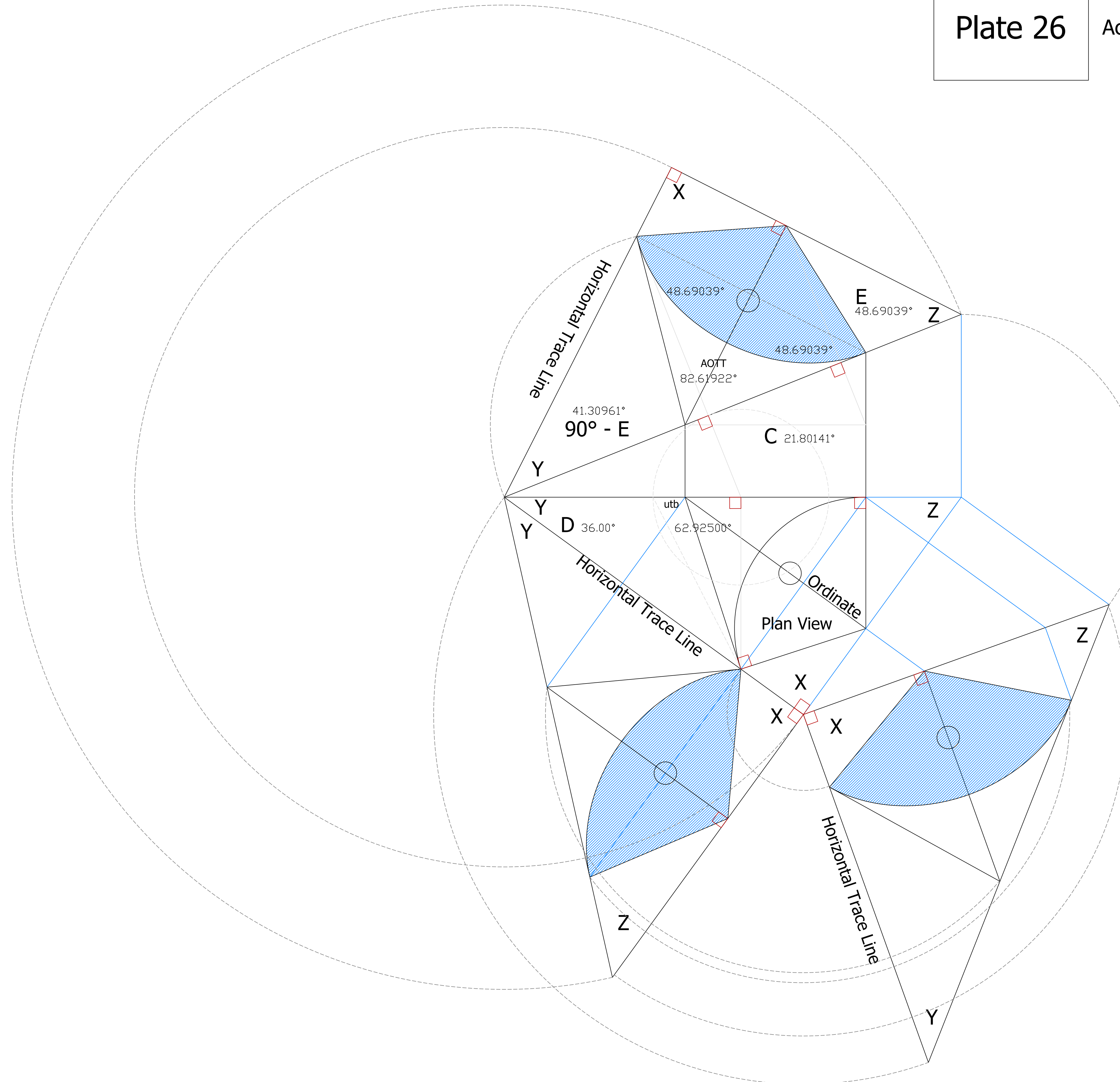
Tetrahedron Angles  
 D Angle = 33.41554  
 A Angle = 51.81525  
 C Angle = 35.00000  
 E Angle = 43.13699  
 B Angle = 41.00242

90-D Angle = 15.41554  
 90-A Angle = 54.00764  
 90-C Angle = 55.00000  
 90-E Angle = 77.42337  
 90-B Angle = 81.01273

90-D Angle = 56.58446  
 90-A Angle = 38.18475  
 90-C Angle = 55.00000  
 90-E Angle = 46.86301  
 90-B Angle = 48.99758

**Plate 26**

Acute Plan with Equally Pitched Tangents



Plan View Section Plane Vertex Information

Plan = Acute  
 Equal Pitched Tangents  
 Plan View Angle = 72.00000  
 Plan View Radius = 8.71851

Tangent Angles

Upper Tangent Angle = 21.80141  
 Lower Tangent Angle = 21.80141  
 Upper Twist Bevel Angle = 62.92500  
 Lower Twist Bevel Angle = 62.92500

Dihedral Angle at Upper Tangent Angle = 62.92500

R1 = 34.23610  
 R4Bm = 48.69039  
 R4Ba = 48.69039  
 Angle of the Tangents =  $180^\circ - 48.69039^\circ - 48.69039^\circ = 82^\circ$

Ellipse Angles

Plan Angle DD = 36.00000  
 Plan Angle D = 36.00000  
 Minor Axis = 8.71851  
 Major Axis = 10.54582  
 Plan View Minor Axis Offset Length = 7.05342  
 Section Plane Minor Axis Offset Length = 8.53175

Tetrahedron Angles

D Angle = 36.00000  
 A Angle = 34.23610  
 C Angle = 21.80141  
 E Angle = 48.69039  
 B Angle = 27.07500

90-D Angle = 54.00000  
 90-A Angle = 55.76390  
 90-C Angle = 68.19859  
 90-E Angle = 41.30961

**Plate 27**

Acute Plan with Shorter Pitched Tangent

Plan View Section Plane Vertex Information

Plan = Acute  
 Unequal Pitched Tangents  
 Plan View Angle = 72.00000  
 Plan View Radius = 8.71851

Tangent Angles  
 Upper Tangent Angle = 35.00000  
 Lower Tangent Angle = 21.80141  
 Upper Twist Bevel Angle = 62.03665  
 Lower Twist Bevel Angle = 51.19183

Dihedral Angle at Upper Tangent Angle = 62.03665

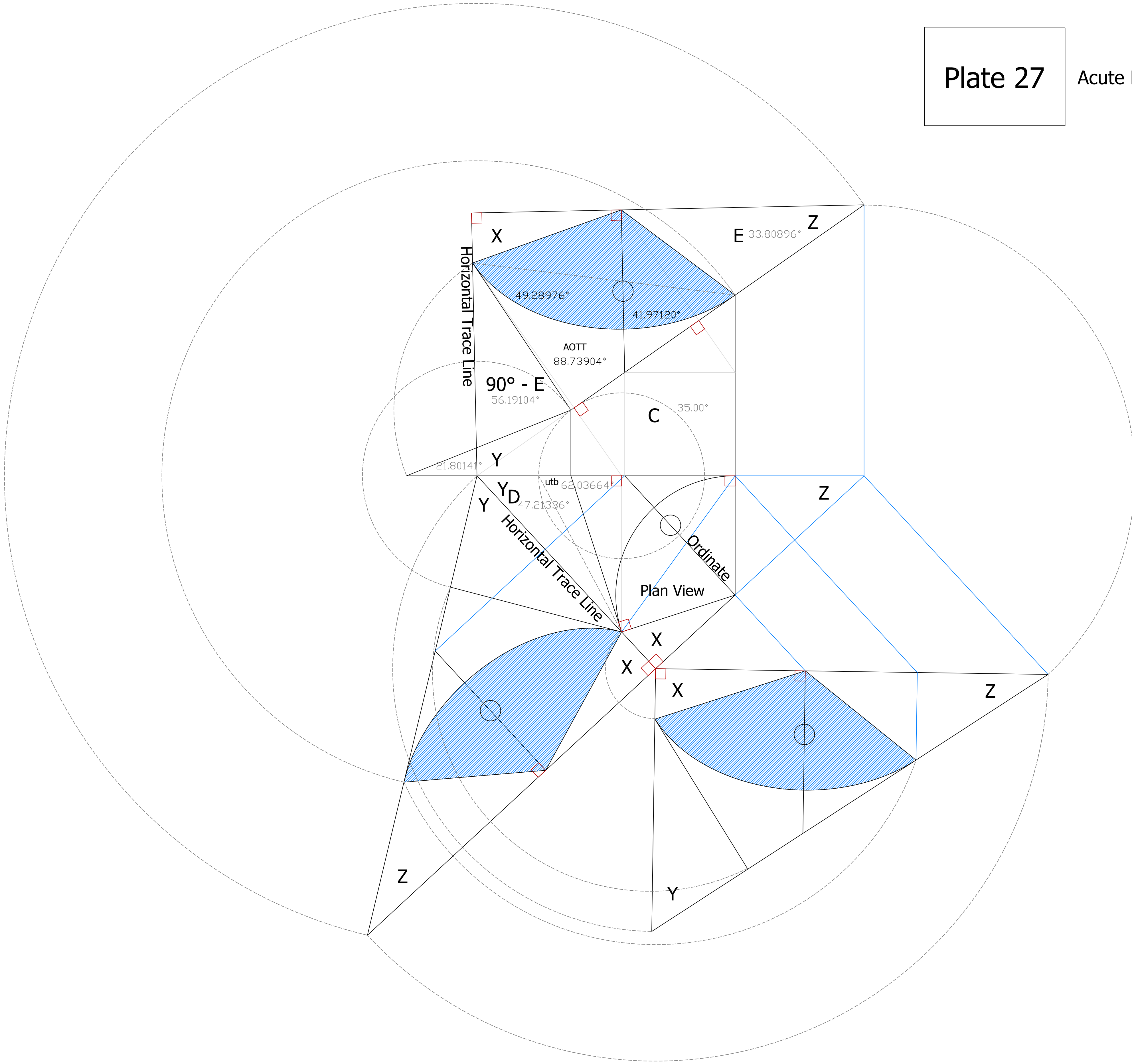
R1 = 43.65462  
 R4Bm = 33.80896  
 R4Ba = 57.45200  
 Angle of the Tangents =  $180^\circ - 33.80896^\circ - 57.45200^\circ = 88.73904^\circ$

Ellipse Angles  
 Plan Angle DD = 47.21336  
 Plan Angle D = 24.78664  
 Minor Axis = 8.71851  
 Major Axis = 12.05023  
 Plan View Minor Axis Offset Length = 7.91532  
 Section Plane Minor Axis Offset Length = 10.94010

Tetrahedron Angles  
 D Angle = 24.78664  
 A Angle = 59.08947  
 C Angle = 35.00000  
 E Angle = 48.04652  
 B Angle = 51.16273

90-D Angle = 65.21336  
 90-A Angle = 30.91053  
 90-C Angle = 55.00000  
 90-E Angle = 41.95348  
 90-B Angle = 38.83727

90-D Angle = 42.78664  
 90-A Angle = 46.34538  
 90-C Angle = 55.00000  
 90-E Angle = 56.19104  
 90-B Angle = 62.03665



### Tangent Handrailing step by step instructions for drawing the face mold pattern for a square plan and equal pitched vertical plane tangents.

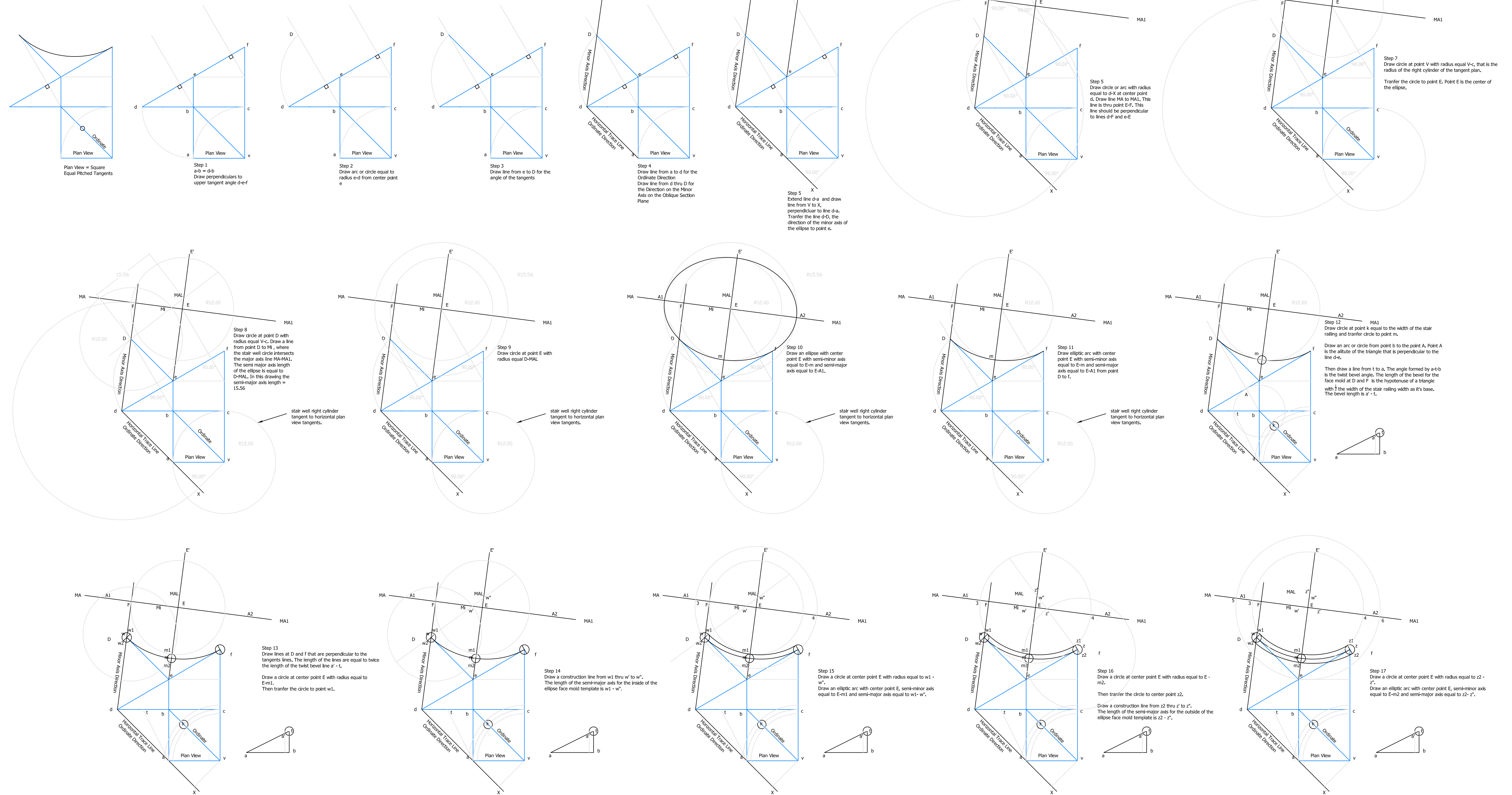
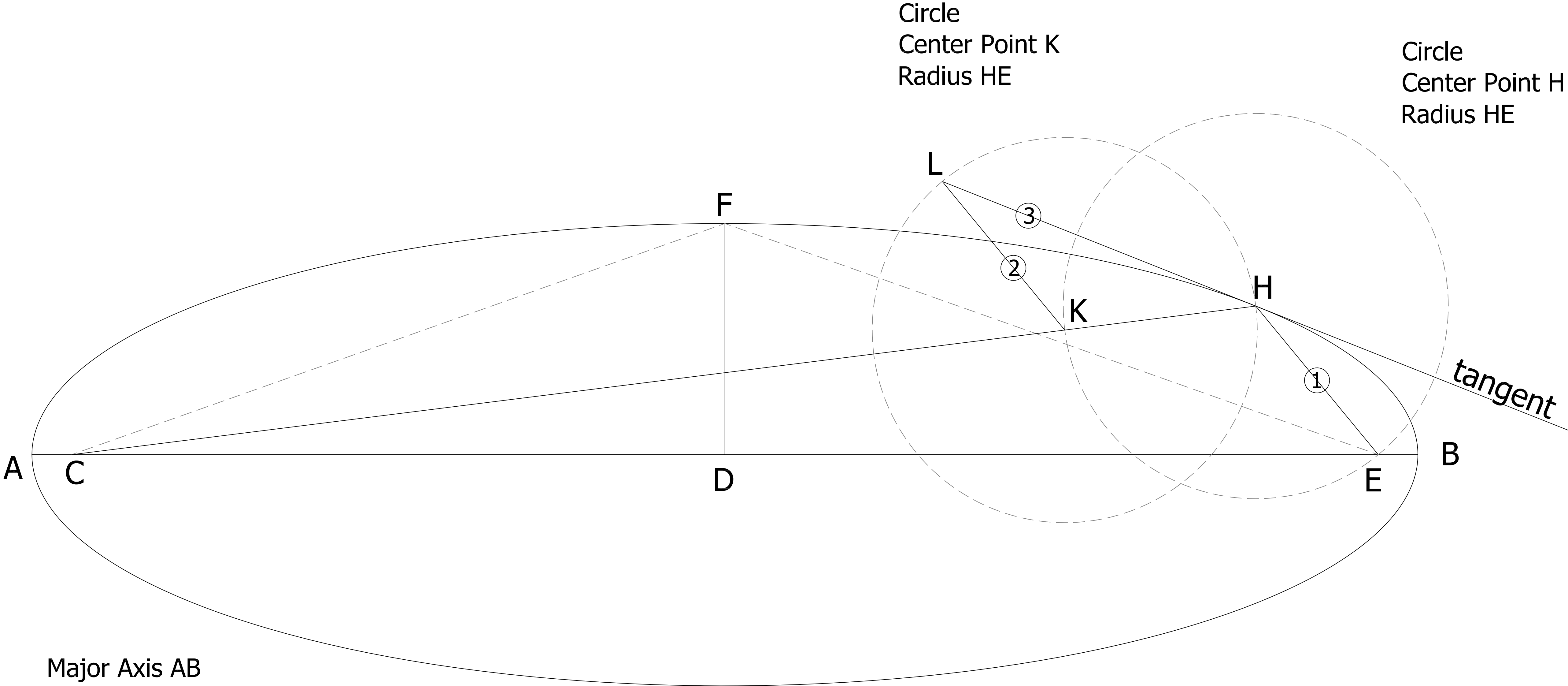


Plate 29

Tangent to an Ellipse  
 from New Elements of Hand-Railing  
 by Robert Riddell 1871



Major Axis AB  
 Minor Axis DF  
 $CF = AD = DB$   
 $EF = AD = DB$   
 Ellipse Foci C & E

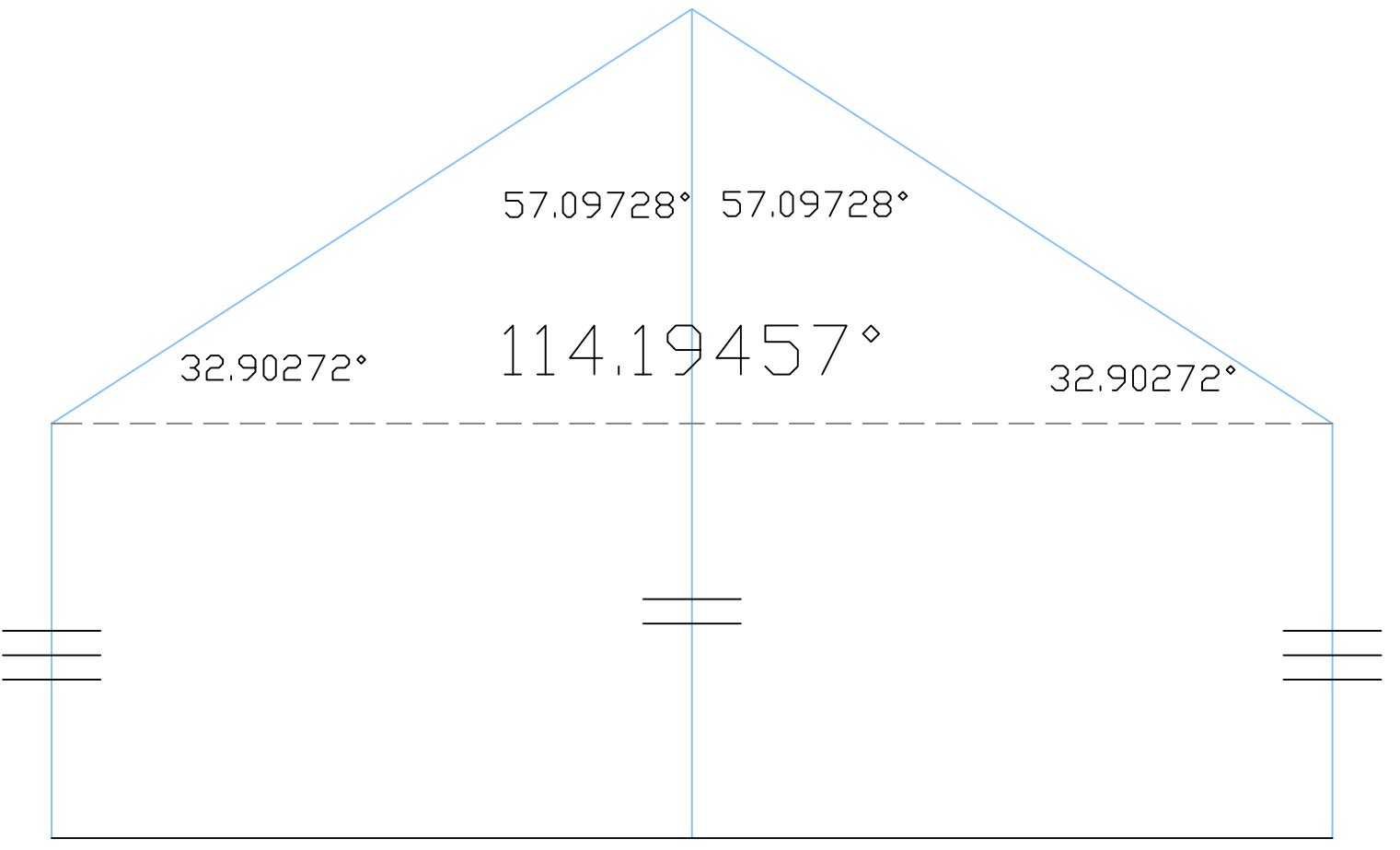
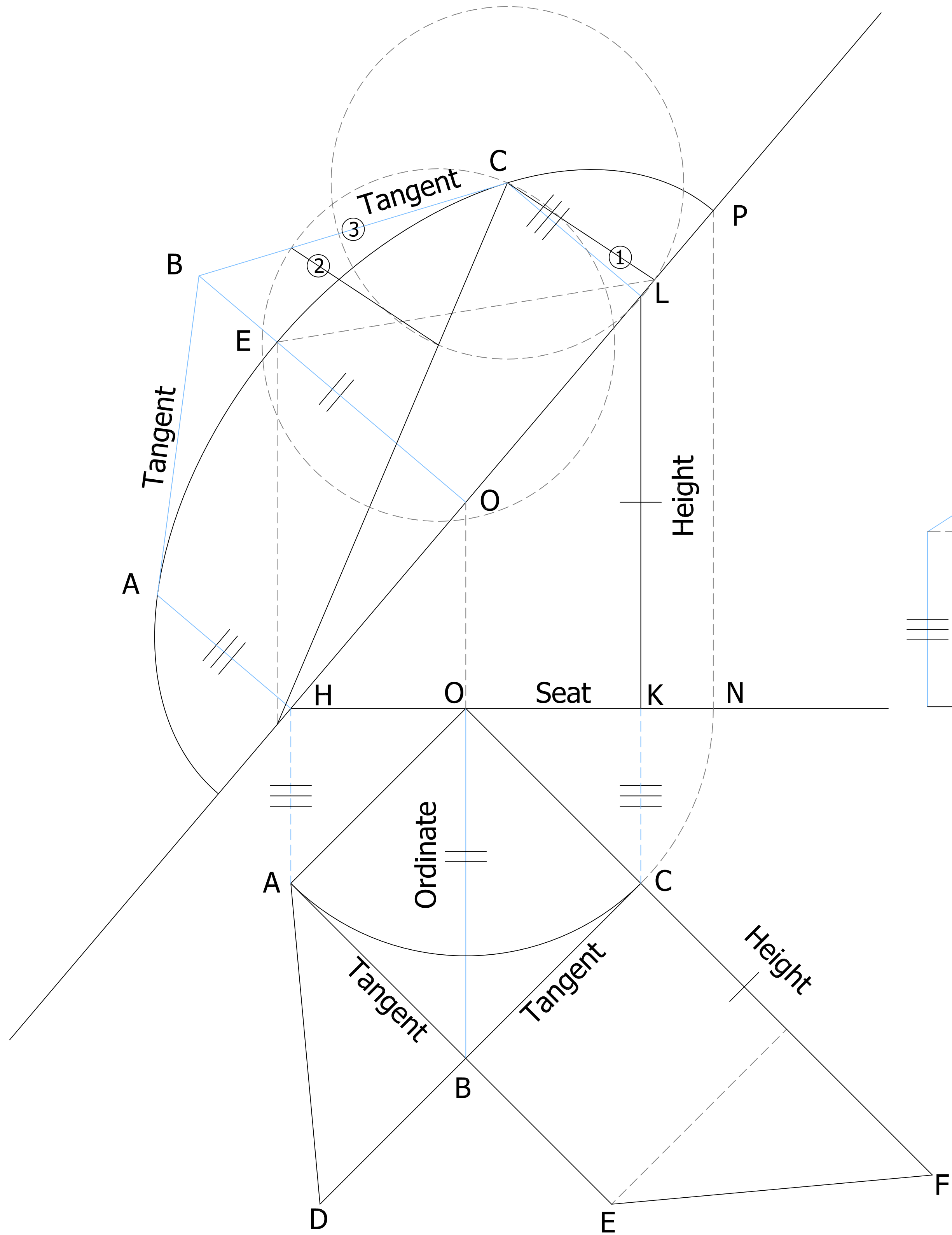
Circle  
 Center Point K  
 Radius HE

Circle  
 Center Point H  
 Radius HE

Lines 1 and 2 are parallel and equal in length  
 Line 3 is tangent to the ellipse at point H

Plate 30

Tangent, Bevel and Elliptic Curve  
from New Elements of Hand-Railing  
by Robert Riddell 1871



Tangent Angles  
 Upper Tangent Angle = 39.80557  
 Lower Tangent Angle = 39.80557  
 Upper Twist Bevel Angle = 57.37326  
 Lower Twist Bevel Angle = 57.37326

Dihedral Angle at Upper Tangent Angle = 57.37326

R1 = 49.68446  
 R4Bm = 32.90273  
 R4Ba = 32.90273  
 Angle of the Tangents =  $180^\circ - 32.90273^\circ - 32.90273^\circ = 114.19454^\circ$

Ellipse Angles  
 Plan Angle DD = 45.00000  
 Plan Angle D = 45.00000  
 Minor Axis = 6.00000  
 Major Axis = 9.27362  
 Plan View Minor Axis Offset Length = 4.24264  
 Section Plane Minor Axis Offset Length = 6.55744  
 Major Axis inside rail width = 7.72802  
 Major Axis outside rail width = 10.81922



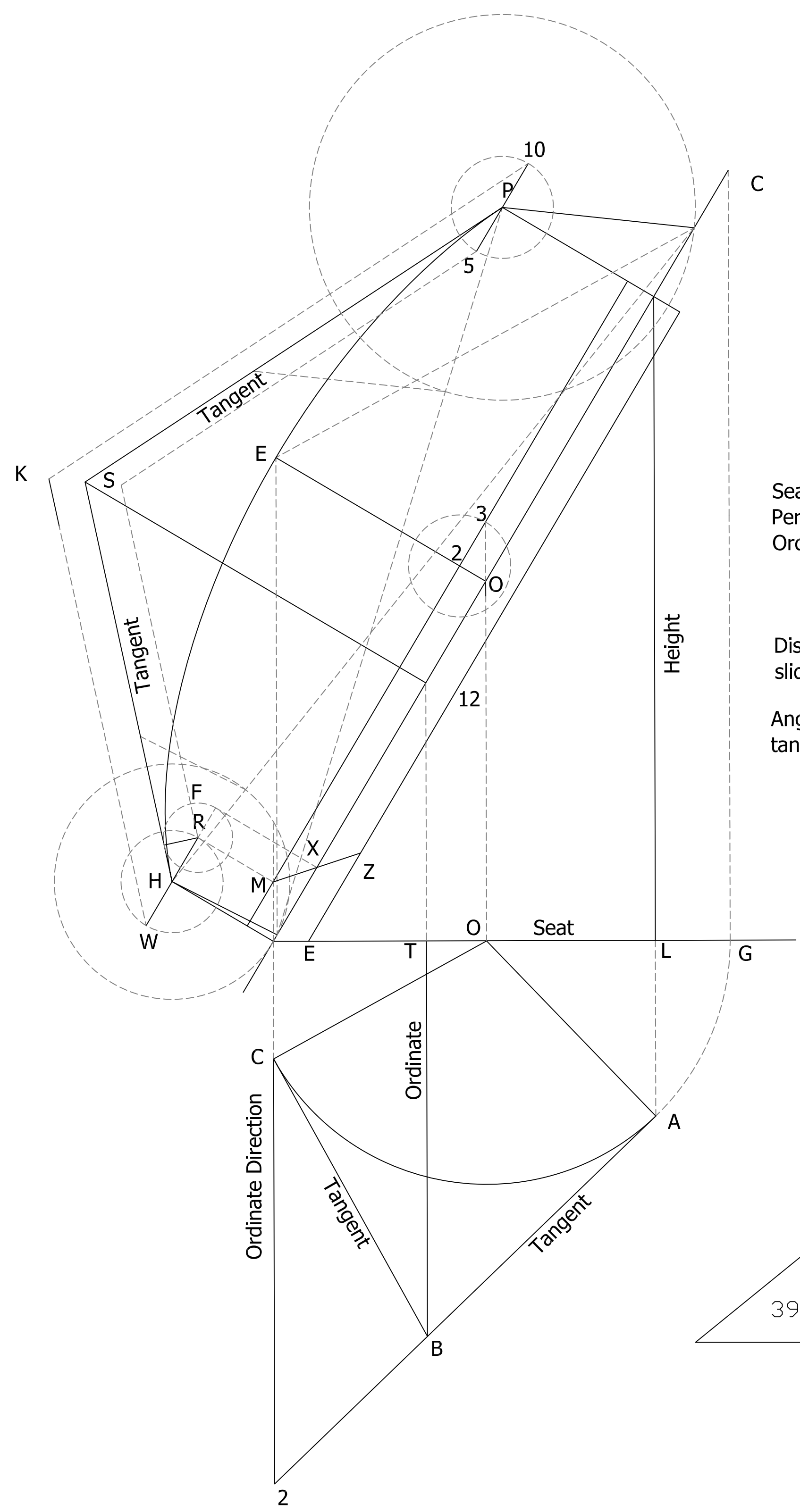


Plate 31

The Ordinate, its Power and value in the Construction of Wreaths  
 from New Elements of Hand-Railing  
 by Robert Riddell 1871

Seat is  
 Perpendicular To  
 Ordinate Direction

Distane 2 - 3 is the length to  
 slide the face mold

Angle M-Z-E is the lower  
 tangent twist bevel angle

Plan View Section Plane Vertex Information  
 Plan View Angle = 75.00000  
 Plan View Radius = 8.04494

Tangent Angles  
 Upper Tangent Angle = 50.56066  
 Lower Tangent Angle = 39.12785  
 Upper Twist Bevel Angle = 53.42673  
 Lower Twist Bevel Angle = 41.12187

Dihedral Angle at Upper Tangent Angle = 53.42673

R1 = 59.32450  
 R4Pm = 26.11051  
 R4Pa = 42.80063  
 Angle of the Tangents =  $180^\circ - 26.11051^\circ - 42.80063^\circ = 111.08886^\circ$

Ellipse Angles  
 Plan Angle DD = 46.14845  
 Plan Angle D = 28.85155  
 Minor Axis = 8.04494  
 Major Axis = 15.76897

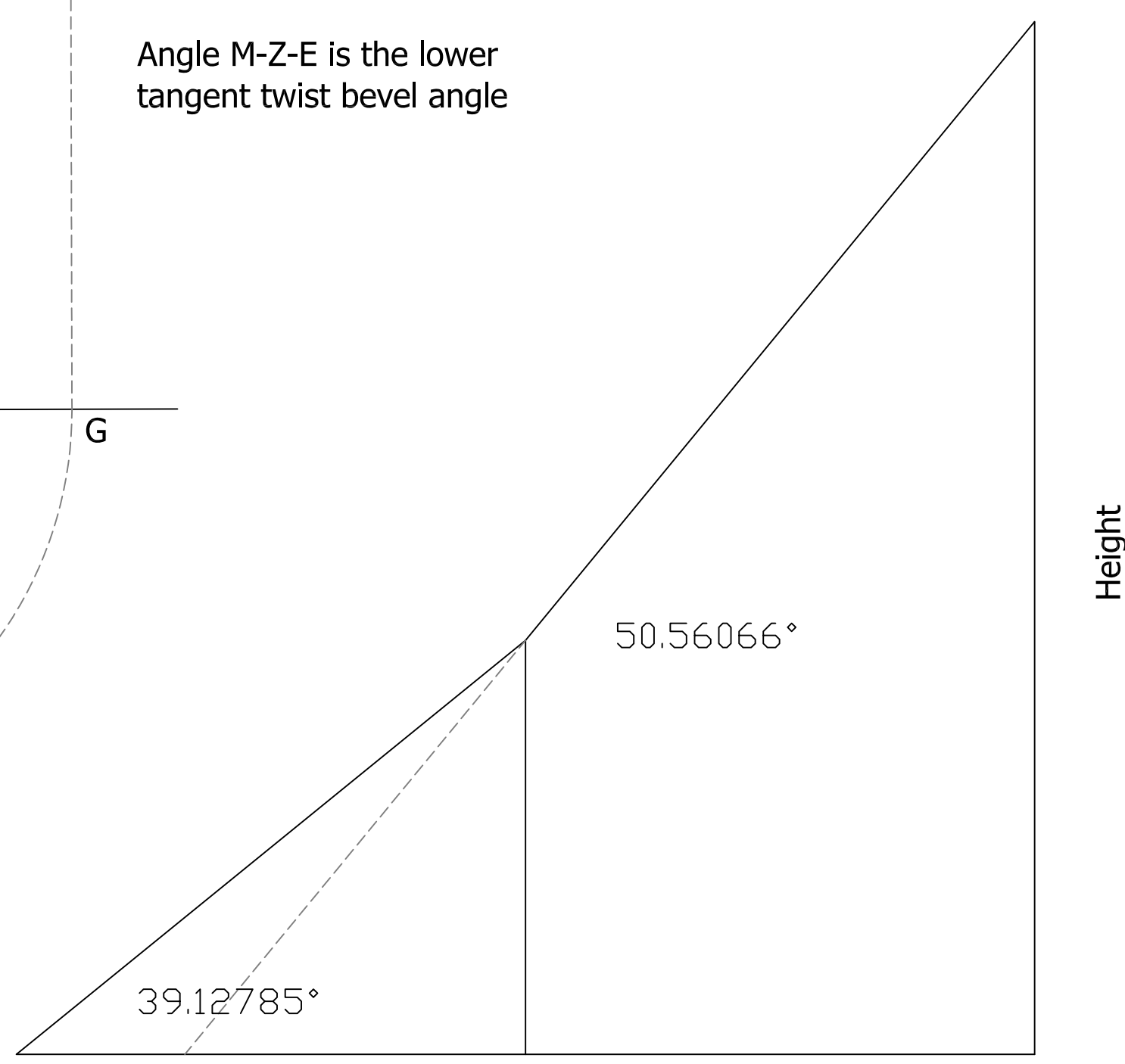
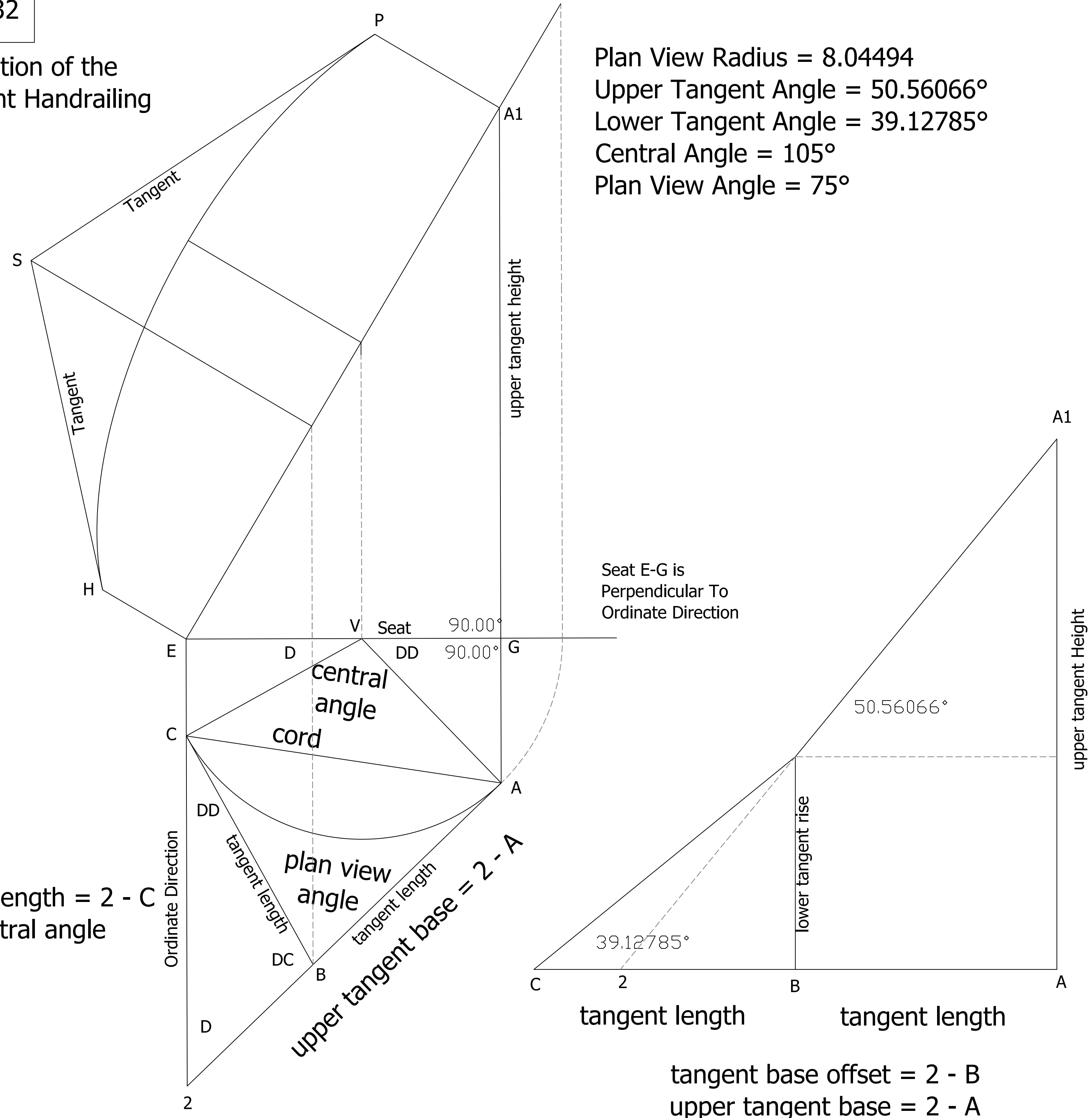


Plate 32

Elucidation of the Tangent Handrailing Angles



Plan View Radius = 8.04494  
 Upper Tangent Angle = 50.56066°  
 Lower Tangent Angle = 39.12785°  
 Central Angle = 105°  
 Plan View Angle = 75°

ordinate length = 2 - C  
 DC = central angle

tangent base offset = 2 - B  
 upper tangent base = 2 - A

cord =  $2 * \text{Radius} * \sin(\text{central angle} \div 2) = 12.76496$   
 tangent length =  $(\text{Radius} * \sin(\text{central angle} \div 2)) \div \cos(\text{central angle} \div 2) = 10.48436$   
 lower tangent rise =  $\text{tangent length} * \tan(\text{lower tangent angle})$   
 lower tangent rise =  $10.48436 * \tan(39.12785) = 8.52888$   
 tangent base offset =  $\text{lower tangent rise} \div \tan(\text{upper tangent angle})$   
 tangent base offset =  $8.52888 \div \tan(50.56066) = 7.015515$   
 upper tangent base =  $\text{tangent length} + \text{tangent base offset}$   
 upper tangent base =  $10.48436 + 7.015515 = 17.49988$   
 upper tangent height =  $\text{upper tangent base} \div \tan(\text{upper tangent angle})$   
 upper tangent height =  $17.49988 * \tan(50.56066) = 21.2749033461$

ordinate length =  $\sqrt{((\text{cord}^2 + \text{upper tangent base}^2) - (2 * \text{cord} * \text{upper tangent base} * \cos(\text{central angle} \div 2)))}$   
 ordinate length =  $\sqrt{((469.1900038160) - (2 * 12.76496 * 17.49988 * \cos(52.5)))}$   
 ordinate length =  $\sqrt{((469.1900038160) - (271.9766701837))} = 14.0432664873$

angle D =  $\arccos((\text{upper tangent base}^2 + \text{ordinate length}^2 - \text{cord}^2) \div (2 * \text{upper tangent base} * \text{ordinate length}))$   
 angle D =  $\arccos((17.49988^2 + 14.0432664873^2 - 12.76496^2) \div (2 * 17.49988 * 14.0432664873))$   
 angle D =  $\arccos((340.515206998) \div (491.511223015)) = 46.148445275$

angle DD =  $180^\circ - \text{central angle} - \text{angle D}$   
 angle DD =  $180^\circ - 105 - 46.148445275 = 28.851554725$

seat =  $(\text{Radius} * \cos(\text{angle DD})) + (\text{Radius} * \cos(\text{angle D}))$   
 seat =  $(8.04494 * \cos(28.851554725)) + (8.04494 * \cos(46.148445275)) = 12.6195363496$

pitch of plank =  $\arctan(\text{upper tangent height} \div \text{seat})$   
 pitch of plank =  $\arctan(21.2749033461 \div 12.6195363496) = 59.3250488281$

semi-major axis =  $\text{Radius} \div \cos(\text{pitch of plank})$   
 semi-major axis =  $8.04494 \div \cos(59.3250488281) = 15.7692298976$